



*Supplement of*

## **Thermal damping and retardation in karst conduits**

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## Details of numerical simulations

Tables S1 and S2 include values for parameters that were different from those provided in

- 5 Table 1 as well as thermal transmission factors and retardation values for all numerical simulations included in Figures 2 and 3. Table S1 provides details about cylindrical simulations, and planar simulations are summarized in Table S2.

**Table S1.** Parameter values used in cylindrical simulations and thermal transmission factors and retardation values of these simulations.

Simulation	Parameter values different from those given in Table 1 (units are the same as those given in Table 1)	F (-)	$\tau$ (s)
C1	$D_H = 0.01$	0.00	-
C2	$D_H = 0.0178$	0.00	-
C3	$D_H = 0.0316$	0.11	16,300
C4	$D_H = 0.0562$	0.43	7,360
C5	$D_H = 0.1$	0.71	3,590
C6	$D_H = 0.178$	0.87	1,840
C7	$D_H = 0.316$	0.94	978
C8	-	0.99	297
C9	$D_H = 3.16$	1.00	93.3
C10	$D_H = 10$	1.00	30.0
C11	$L = 100$	1.00	29.9
C12	$L = 316$	1.00	96.5
C13	$L = 3,160$	0.96	952
C14	$L = 10,000$	0.87	2,990
C15	$L = 31,600$	0.64	9,700
C16	$V = 0.00626$	0.31	15,700
C17	$V = 0.0111$	0.48	11,900
C18	$V = 0.0198$	0.65	7,830
C19	$V = 0.0352$	0.78	4,770
C20	$V = 0.0626$	0.87	2,830
C21	$V = 0.198$	0.96	924
C22	$V = 1.98$	1.00	95.2
C23	$V = 6.26$	1.00	30.1
C24	$\mathcal{R}_D = 1,900$	0.94	51.4
C25	$\mathcal{R}_D = 6,000$	0.96	92.5
C26	$\mathcal{R}_D = 19,000$	0.98	166

**Table S1.** Continued.

Simulation	Parameter values different from those given in Table 1 (units are the same as those given in Table 1)	$F$ (-)	$\tau$ (s)
C27	$\mathcal{R}_D = 190,000$	0.99	538
C28	$\mathcal{R}_D = 600,000$	0.99	983
C29	$\mathcal{R}_A = 1$	0.99	297
C30	$\mathcal{R}_A = 3.16$	0.99	298
C31	$\mathcal{R}_A = 31.6$	0.99	298
C32	$\mathcal{R}_A = 100$	0.99	297
C33	$k_r = 1$	0.99	203
C34	$k_r = 1.5$	0.99	248
C35	$k_r = 2$	0.99	286
C36	$k_r = 2.5$	0.98	320
C37	$k_r = 3$	0.98	351
C38	$c_{p,r} = 700$	0.99	276
C39	$c_{p,r} = 800$	0.99	295
C40	$c_{p,r} = 900$	0.99	313
C41	$c_{p,r} = 1,000$	0.98	330
C42	$\rho_r = 2,000$	0.99	276
C43	$\rho_r = 2,250$	0.99	292
C44	$\rho_r = 2,500$	0.99	308
C45	$\rho_r = 2,750$	0.98	323
C46	$\rho_r = 3,000$	0.98	337
C47	$k_w = 0.5$	0.99	297
C48	$k_w = 0.55$	0.99	297
C49	$k_w = 0.6$	0.99	297
C50	$k_w = 0.65$	0.99	297
C51	$k_w = 0.7$	0.99	297
C52	$c_{p,w} = 4,100$	0.99	304

**Table S1.** Continued.

Simulation	Parameter values different from those given in Table 1 (units are the same as those given in Table 1)	$F$ (-)	$\tau$ (s)
C53	$c_{p,w} = 4,150$	0.99	300
C54	$c_{p,w} = 4,250$	0.99	293
C55	$c_{p,w} = 4,300$	0.99	290
C56	$\rho_w = 980$	0.99	303
C57	$\rho_w = 990$	0.99	300
C58	$\rho_w = 1,010$	0.99	294
C59	$\rho_w = 1,020$	0.99	291
C60	$\mu_w = 0.0006$	0.99	298
C61	$\mu_w = 0.0009$	0.99	297
C62	$\mu_w = 0.0012$	0.99	297
C63	$\mu_w = 0.0015$	0.99	297
C64	$\mu_w = 0.0018$	0.99	296
C65	$D_H = 0.1; V = 0.0346$	0.00	-
C66	$D_H = 10; V = 0.907$	1.00	20.5
C67	$D_H = 0.1; V = 0.110$	0.15	21,400
C68	$D_H = 10; V = 2.87$	1.00	6.66
C69	$D_H = 0.1; V = 0.346$	0.54	6,530
C70	$L = 100; V = 0.198$	1.00	92.9
C71	$L = 10,000; V = 0.198$	0.64	9,530
C72	$L = 100; V = 1.98$	1.00	9.31
C73	$L = 10,000; V = 1.98$	0.96	946
C74	$V = 0.198; \mathcal{R}_D = 6,000$	0.89	283
C75	$V = 0.198; \mathcal{R}_D = 600,000$	0.98	3,090
C76	$V = 1.98; \mathcal{R}_D = 6,000$	0.99	29.5
C77	$V = 1.98; \mathcal{R}_D = 600,000$	1.00	295
C78	$V = 0.198; \mathcal{R}_A = 1$	0.96	930

**Table S1.** Continued.

Simulation	Parameter values different from those given in Table 1 (units are the same as those given in Table 1)	$F$ (-)	$\tau$ (s)
C79	$V = 0.198; \mathcal{R}_A = 100$	0.96	930
C80	$V = 1.98; \mathcal{R}_A = 1$	1.00	94.8
C81	$V = 1.98; \mathcal{R}_A = 100$	1.00	94.5
C82	$D_H = 0.316; V = 0.273$	0.87	2,240
C83	$L = 20,000$	0.75	6,040
C84	$L = 100; V = 0.0626$	0.99	289
C85	$L = 10,000; V = 6.26$	0.99	300
C86	$D_H = 0.05; V = 0.0586; \mathcal{R}_D = 6,000,000$	0.00	-
C87	$L = 200,000; \mathcal{R}_D = 6,000,000$	0.51	733,000
C88	$D_H = 0.2; V = 0.0609; \mathcal{R}_D = 6,000,000$	0.61	287,000
C89	$L = 10,000; \mathcal{R}_D = 6,000,000$	0.97	35,600
C90	$L = 10,000; V = 0.198; \mathcal{R}_D = 6,000,000$	0.90	113,000
C91	$D_H = 0.02; V = 0.213; \mathcal{R}_D = 6,000,000$	0.00	-
C92	$D_H = 0.632; L = 3,430; V = 0.231; \mathcal{R}_D = 6,000,000$	0.04	1,560,000
C93	$D_H = 0.2; L = 28,600; V = 0.609; \mathcal{R}_D = 6,000,000$	0.24	871,000
C94	$D_H = 0.02; L = 100; V = 0.213; \mathcal{R}_D = 600$	0.04	559
C95	$D_H = 0.0356; L = 110; V = 0.131; \mathcal{R}_D = 600$	0.08	478
C96	$D_H = 0.0632; L = 343; V = 0.231; \mathcal{R}_D = 600$	0.12	500
C97	$D_H = 0.02; L = 100; V = 0.213$	0.25	8,440
C98	$D_H = 0.0632; L = 343; V = 0.231$	0.52	5,790
C99	$D_H = 0.02; V = 0.213$	0.00	-
C100	$D_H = 0.0632; L = 3,430; V = 0.231$	0.00	-
C101	$D_H = 0.2; L = 28,600; V = 0.609$	0.04	61,200
C102	$D_H = 10; L = 20,000; V = 0.287$	0.95	1,280
C103	$D_H = 0.3; L = 20,000; V = 0.263; \mathcal{R}_D = 6,000,000$	0.32	802,000
C104	$D_H = 0.5; L = 20,000; V = 0.383; \mathcal{R}_D = 6,000,000$	0.71	275,000

**Table S2.** Parameter values used in planar simulations and thermal transmission factors and retardation values of these simulations.

Simulation	Parameter values different from those given in Table 1 (units are the same as those given in Table 1)	$F$ (-)	$\tau$ (s)	
P1 (equivalent to C1)	$D_H = 0.01$	0.35	33,200	
P2 (equivalent to C2)	$D_H = 0.0178$	0.55	17,700	
P3 (equivalent to C3)	$D_H = 0.0316$	0.71	9,670	
P4 (equivalent to C4)	$D_H = 0.0562$	0.82	5,360	
P5 (equivalent to C5)	$D_H = 0.1$	0.90	2,990	
P6 (equivalent to C6)	$D_H = 0.178$	0.94	1,670	
P7 (equivalent to C7)	$D_H = 0.316$	0.97	935	
P8 (equivalent to C65)	$D_H = 0.1; V = 0.0346$	0.17	63,900	
P9 (equivalent to C67)	$D_H = 0.1; V = 0.110$	0.54	17,700	
P10 (equivalent to C69)	$D_H = 0.1; V = 0.346$	0.82	5,430	
P11 (equivalent to C82)	$D_H = 0.316; V = 0.273$	0.92	2,140	
P12 (equivalent to C86)	$D_H = 0.05; V = 0.0586; \mathcal{R}_D = 6,000,000$	0.79	645,000	
P13 (equivalent to C88)	$D_H = 0.2; V = 0.0609; \mathcal{R}_D = 6,000,000$	0.94	153,000	
P14 (equivalent to C91)	$D_H = 0.02; V = 0.213; \mathcal{R}_D = 6,000,000$	0.85	443,000	
P15 (equivalent to C92)	$D_H = 0.0632; L = 3,430; V = 0.231; \mathcal{R}_D = 6,000,000$	0.85	443,000	
P16 (equivalent to C93)	$D_H = 0.2; L = 28,600; V = 0.609; \mathcal{R}_D = 6,000,000$	0.85	443,000	
P17 (equivalent to C94)	$D_H = 0.02; L = 100; V = 0.213; \mathcal{R}_D = 600$	0.22	503	
P18 (equivalent to C95)	$D_H = 0.0356; L = 110; V = 0.131; \mathcal{R}_D = 600$	0.22	476	
P19 (equivalent to C96)	$D_H = 0.0632; L = 343; V = 0.231; \mathcal{R}_D = 600$	0.22	495	
P20 (equivalent to C97)	$D_H = 0.02; L = 100; V = 0.213$	0.85	4,410	
P21 (equivalent to C98)	$D_H = 0.0632; L = 343; V = 0.231$	0.85	4,400	
P22 (equivalent to C99)	$D_H = 0.02; V = 0.213$	0.22	52,200	
P23 (equivalent to C100)	$D_H = 0.0632; L = 3,430; V = 0.231$	0.22	52,100	
P24 (equivalent to C101)	$D_H = 0.2; L = 28,600; V = 0.609$	0.22	52,300	
P25 (equivalent to C103)	$D_H = 0.3; L = 20,000; V = 0.263; \mathcal{R}_D = 6,000,000$	0.84	478,000	
P26 (equivalent to C104)	$D_H = 0.5; L = 20,000; V = 0.383; \mathcal{R}_D = 6,000,000$	0.93	195,000	