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Over the recent years, a significant progress in the field of Model Predictive Control (MPC) has been achieved. The purpose of the *International Symposium on Advanced Model Predictive Control* is to consider some of the latest achievements in this field. The symposium covers three main topical areas: MPC theory, computational aspects of MPC and MPC applications.

In the paper “Stability of TITO predictive functional control systems”, a Two Input-Two Output (TITO) interconnected linear plant with first order plus time delay transfer functions is investigated. The basic algorithm for unconstrained Predictive Functional Control (PFC) is presented. The stability conditions are derived and represented in an explicit form as a multivariable function of the generalized tuning parameters of PFC controller.

The paper “Explicit approximate nonlinear predictive control based on neural network models” suggests an approximate multi-parametric Nonlinear Programming approach to explicit solution of nonlinear MPC problems for constrained nonlinear systems based on neural network models. In particular, the reference tracking problem is considered.

In the paper “Fuzzy model predictive control algorithm, case study”, a method for design of nonlinear predictive controller based on a fuzzy model is presented. The Takagi-Sugeno fuzzy model with neural-fuzzy implementation is used and incorporated into the predictive controller. An on-line optimization approach with simplified gradient technique is proposed to calculate the future control actions.

The paper “Stochastic predictive control of a thermoelectric power plant” considers the application of an on-line optimization approach for stochastic nonlinear MPC to the reference tracking control of a combustion plant. The controller brings the air factor on its optimal value with every change of the load factor and thus an optimal operation of the combustion plant is achieved.

The paper “MPC approach in production control” suggests an MPC approach to develop production control systems. Within this approach, different production Key Performance Indicators are identified which are used as referenced controlled variables. Then, an MPC controller is designed to control the production process. The proposed method is applied for the closed-loop control at the production-management level of a polymerization plant.

In the paper “Experience of MPC application in power plants”, the experience gained from application of different MPC approaches in a number of Bulgarian thermal power plants, simulation based on real model and some practical applications are presented. The coordinated control of output power and inlet turbine steam pressure are considered as well as the design of the main local control systems – primary superheater temperature, mill-fan pulverization system, combustion process.

The paper “Coordinated model predictive control of a power plant” compares the control performance of the most popular predictive control strategies – Dynamic Matrix Control and Predictive Functional Control. The developed algorithms have been tested on a two input – two output non-linear boiler-turbine model of a power plant at equal conditions. Both of them proved to be among the best control strategies showing good control performance and robustness.

In the paper, “Fuzzy-neural model predictive control algorithm for temperature control of polymer reactor mixture”, an algorithm for fuzzy-neural model predictive control is presented. The algorithm is applied for temperature control of the reaction mixture in a polymer reactor. The results with different references show that the proposed control algorithm is applicable for polymerization processes with highly nonlinear characteristics.

The paper “Advanced process control project: A brief overview”, considers an Advanced Process Control (APC) technology which aims at process optimization and control performance improvement by implementing MPC algorithms. The features of the APC technology and project implementation are overviewed based on the case studies of APC application at crude distillation units of Russian refineries.
STOCHASTIC PREDICTIVE CONTROL OF A THERMOELECTRIC POWER PLANT

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Abstract: Energy production is one of the largest sources of air pollution. A feasible method to reduce the harmful flue gas emissions and to increase the efficiency is to improve the control strategies of the existing thermoelectric power plants. This makes the Nonlinear Model Predictive Control (NMPC) method very suitable for achieving an efficient combustion control. Recently, an approach for stochastic NMPC based on a Gaussian process model was proposed. In this paper, an on-line optimization approach for stochastic NMPC is applied to the reference tracking control of a combustion plant based on its Gaussian process model. The controller brings the air factor (respectively the concentration of oxygen in the flue gas) on its optimal value with every change of the load factor and thus an optimal operation of the combustion plant is achieved.

Key words: Model Predictive Control, Stochastic Systems, Probabilistic Models, Power Plants

INTRODUCTION

Energy production is one of the largest sources of air pollution. Therefore a rational and ecological use of energy is the main task of the thermoelectric power plants. A feasible method to reduce the NOx, CO, CO2 emissions and to increase the efficiency is to improve the control strategies of existing power plants, i.e. to optimize the combustion process [1]. The objectives for the improvement of the power plant combustion process are energy saving, pollution reduction, longer plant lifetime, less downtime and maintenance effort, increased safety in operation, i.e. overall cost reduction. These goals can be achieved through application of modern control algorithms. Feedback combustion control is possible since continuous flue gas analyzers are available [1]. For control purposes it would be ideal to measure all flue gas components. But the price for such realization would be too high in comparison with the savings achieved. Therefore the control of the oxygen fraction in the flue gas, measured on-line by the well known in-situ ZrO2 analyzers, is often the best solution [2]. Based on that, different algorithms for combustion control have been studied in [2], [3], [4], [5], [6] and more recent in e.g. [7]. It should be noted that these methods assume that the combustion model is known exactly. However, the mathematical models are only an approximation of the real process and they usually contain some amount of uncertainty (unknown additive disturbances and/or uncertain model parameters). In order to achieve a robust performance of the control system it would be necessary to take into account the uncertainty when designing the controller.

Nonlinear Model Predictive Control (NMPC) has become the accepted methodology to solve complex control problems related to process industries [8]. It involves the solution at each sampling instant of a finite horizon optimal control problem subject to nonlinear system dynamics and state and input constraints. Stochastic NMPC problems are formulated in the applications where the system to be controlled is described by a stochastic model. Thus, the approaches in [9], [10], [11] are based on linear state space models with stochastic parameters and/or additive noise and they optimize the expected value of the cost function subject to hard input constraints [9] or probabilistic constraints [10], [11]. In [12], [13], [14], stochastic MPC approaches incorporating a probabilistic cost and probabilistic constraints are developed. The method suggested in [12] is based on a moving average (MA) model with random coefficients. It was further extended to linear time-varying MA models [13] and to state space models with stochastic uncertainty in the output or the input map [14]. It should be noted that the mentioned stochastic MPC approaches are based on parametric probabilistic models. Alternatively, the stochastic systems can be modeled with non-parametric models which can offer a significant advantage compared to the parametric models. This is related to the fact that the non-parametric probabilistic models provide information about prediction uncertainties which are difficult to evaluate appropriately with the parametric models. The Gaussian process model is an example of a non-parametric probabilistic black-box model and up to now it has been applied to model mainly static nonlinearities. The use of Gaussian processes in the modelling of dynamic systems is a recent development e.g. [15], [16]. In [17], [18], [19], an on-line optimization approach for stochastic NMPC based on Gaussian process model is proposed.

In this paper, a Gaussian process model of a combustion plant is obtained. Then, an on-line optimization approach for stochastic NMPC is applied to the reference tracking control of the combustion plant, where the goal is to bring the air factor on its optimal value with every change of the load factor.

The following abbreviation and notation will be used in the paper. The nonlinear model predictive control problem based on Gaussian process model will be referred to as GP-NMPC problem. \( A > 0 \) means that the square matrix \( A \) is positive definite. For \( x \in \mathbb{R}^n \), the weighted norm is defined for some
symmetric matrix \( A \succ 0 \) as \( \|A\|_2 = \sqrt{\text{tr}\{A^T A\}} \). For a random variable \( y \) with Gaussian distribution, \( \mathcal{N}(\mu(y), \sigma^2(y)) \) denotes its probability distribution, and \( \mu(y) \) and \( \sigma^2(y) \) are respectively its mean and variance.

**GAUSSIAN PROCESS MODEL OF A COMBUSTION PLANT**

Modelling of dynamic systems with Gaussian processes. A Gaussian process is an example of the use of a flexible, probabilistic, nonparametric model which directly provides us with uncertainty predictions. Its use and properties for modelling are reviewed in [20].

A Gaussian process is a collection of random variables which have a joint multivariate Gaussian distribution. Assuming a relationship of the form \( y = f(z) \) between an input \( z \in \mathbb{R}^D \) and output \( y \in \mathbb{R} \), we have \( y(1), y(2), \ldots, y(M) \sim \mathcal{N}(0, \Sigma) \), where \( \Sigma = \text{Cov}(y(p), y(q)) = C(z(p), z(q)) \) gives the covariance between the output points \( y(p) \) and \( y(q) \) corresponding to the input points \( z(p) \) and \( z(q) \). Thus, the mean \( \mu(z) \) (usually assumed to be zero) and the covariance function \( C(z(p), z(q)) \) fully specify the Gaussian process.

Note that the covariance function \( C(z(p), z(q)) \) can be any function with the property that it generates a positive definite covariance matrix. A common choice is:

\[
C(z(p), z(q)) = \exp\left[ -\frac{1}{2} \sum_{i=1}^{D} w_i (z_i(p) - z_i(q))^2 \right] + \sigma_p^2 \delta_{pq}
\]

where \( \Theta = [w_1, \ldots, w_D, \sigma_p, \sigma_q] \) are the ‘hyperparameters’ of the covariance function, \( z_i \) denotes the \( i \)-th component of the \( D \)-dimensional input vector \( z \), and \( \sigma_{pq} \) is the Kronecker operator. For a given problem, the hyperparameters are learned (identified) using the data at hand. Consider a set of \( M \) \( D \)-dimensional input vectors \( Z = [z(1), z(2), \ldots, z(M)] \) and a vector of output data \( Y = [y(1), y(2), \ldots, y(M)] \). Based on the data set \((Z, Y)\), the parameters of the covariance function are tuned by maximizing their log-likelihood, which is computationally relatively demanding since the inverse of the data covariance matrix of size \( M \times M \) has to be calculated at every iteration.

The obtained Gaussian process model can be used for regression calculation, i.e. to estimate the probability distribution of an output \( y' \) which corresponds to a new input vector \( z' \). For a new test input \( z' \), the predictive distribution of the corresponding output is \( y' | z', Z, Y \) and is Gaussian, with mean and variance:

\[
\begin{align*}
\mu(z') &= k(z')^T \mathbf{K}^{-1} \mathbf{y} \\
\sigma^2(z') &= k(z') - k(z')^T \mathbf{K}^{-1} k(z') + \sigma_p^2
\end{align*}
\]

Here, \( \mathbf{K} \) is the covariance matrix of size \( M \times M \) corresponding to the training set \( Z \), \( k(z') = [C(z(1), z'), \ldots, C(z(M), z')]^T \) is the \( M \times 1 \) vector of covariances between the test and training cases, and \( k(z') = C(z', z') \) is the covariance between the test input and itself.

Gaussian processes can be used to model static nonlinearities and can therefore be used for modelling of dynamic systems if delayed input and output signals are used as regressors [15]. In such cases an autoregressive model is considered, such that the current predicted output depends on previous estimated outputs, as well as on previous control inputs:

\[
z(t) = [\hat{y}(t-1), \hat{y}(t-2), \ldots, \hat{y}(t-L), u(t-1), u(t-2), \ldots, u(t-L)]^T
\]

where \( \hat{y}(t) = f(z(i)) + \eta(t) \)

\[y(t) = f(z(i)) + \eta(t)\]

where \( t \) denotes consecutive number of data sample, \( L \) is a given lag, and \( \eta(t) \) is the prediction error. The quality of the predictions with a Gaussian process model is assessed by computing the average squared error (ASE):

\[
ASE = \frac{1}{M} \sum_{i=1}^{M} \{\mu(\hat{y}(i)) - y(i)\}^2
\]

and by the log density error (LD) [15]:

\[
LD = \frac{1}{2M} \sum_{i=1}^{M} \log(2\pi) + \log(\sigma^2(\hat{y}(i))) + \frac{[\mu(\hat{y}(i)) - y(i)]^2}{\sigma^2(\hat{y}(i))}
\]

In (4), (5), \( \mu(\hat{y}(i)) \) and \( \sigma^2(\hat{y}(i)) \) are the prediction mean and variance, \( y(i) \) is the system’s output and \( M \) is the number of the training points.

**Gaussian process model of a combustion plant.** The system under investigation is a process of combustion in a steam boiler PK 401 at Cinkarna Celje Company, Celje, Slovenia. In order to understand the basic relations among variables and to illustrate the non-linearity of the combustion process, a mathematical model is introduced (the derivation of the mathematical model can be found in [21]):

\[
\frac{dx_t}{dt} = \frac{1}{V_k} x_t (\Phi_{\text{ar}} + \Phi_{\text{fuel}} (V_d - V_f)) + 2 \Phi_{\text{ar}} - 100 \Phi_{\text{fuel}}
\]

In (6), \( x_{o_2} \) is the percentage of \( O_2 \) [vol\%] in the flue gases, \( V_f \) is the volume of the combustion chamber [m\(^3\)], \( \Phi_{\text{ar}} \) is the normalized total flow of fuel [kg s\(^{-1}\)], \( \Phi_{\text{fuel}} \) is the normalized total flow of air [Nm\(^3\) kg\(^{-1}\)], \( V_d \) is the theoretically required oxygen volume for the combustion of one unit of fuel [Nm\(^3\) kg\(^{-1}\)], \( V_f \) is the theoretically obtained gas volume from one unit of fuel [Nm\(^3\) kg\(^{-1}\)]. The air flow \( \Phi_{\text{ar}} \) is controlled through a damper. As it is part of the closed-loop, it has to be modeled and added to the combustion model. The dependence of the air flow \( \Phi_{\text{ar}} \) on the angle \( \phi \) of the damper is given by the following relation [4]:

\[
\Phi_{\text{ar}} = \Phi_{\text{ar,max}} \frac{3(\phi - 45)}{45}, \quad 0^\circ \leq \phi \leq 45^\circ
\]

\[
\Phi_{\text{ar}} = \Phi_{\text{ar,max}} \left[ 2 - \frac{3(\phi - 45)}{45} \right], \quad 45^\circ \leq \phi \leq 90^\circ
\]

where \( \Phi_{\text{ar,max}} \) is the maximum flow of air.

The sampling time, determined according to system dynamics, was selected to be \( T_s = 1 \text{ s} \). Then, the dynamics of \( x_{o_2} \) can be represented by the following nonlinear discrete-time model:

\[x_{o_2}(t+1) = f(x_{o_2}(t), \Phi_{\text{fuel}}(t), \phi(t)) + \xi(t)
\]

where \( \xi(t) \in \mathbb{R} \) is Gaussian disturbance which represents the additive effect of the unmeasured stochastic disturbances (e.g. change in the fuel composition).

The signals \( \phi \) and \( \Phi_{\text{fuel}} \) for identification are generated by random number generators with normal distributions and the signal \( x_{o_2} \) is computed from equation (6). The rates of change of the signals \( \phi \) and \( \Phi_{\text{fuel}} \) are respectively \( T_s = 5 \text{ s} \) and \( T_{\text{fuel}} = 100 \text{ s} \). The number of the signals samples used for the identification is \( M = 1000 \). A Gaussian disturbance \( \xi \) with zero mean and variance 0.05 is used. Based on the generated data set, the discrete-time system (9) is
approximated with Gaussian process with the following hyperparameters:
\[
\Theta = [w_1, w_2, w_3, v_1, v_2]
\]
\[
= [0.01346, 0.02847, 0.00036, 0.21984, 0.55, 0.56554]
\] (10)
The maximum likelihood framework is used to determine the hyperparameters. The optimization method applied for identification of the Gaussian process model is the conjugate gradient method with line searches [22]. The prediction error associated to the identification signal is \(ASE = 0.6051\). The signals \(\phi\) and \(\Phi_{\text{opt}}\) used for validation have rates of change which are different from those of the identification signals. The prediction error corresponding to the validation signal is \(ASE = 0.9177\).

### STOCHASTIC PREDICTIVE CONTROL OF THE COMBUSTION PLANT

#### Optimal operation of combustion plants
The limited fuel sources, considerable increase in the fuel prices and the enormous environment pollution require decreasing the fuel use, the heat losses and the amount of harmful flue-gas emissions, i.e. to optimize the combustion process [4]. It has been shown in [4] that in order to achieve an optimal operation of the combustion plants, it is necessary to optimize the air factor \(\lambda\) defined as:
\[
\lambda = \frac{V_{\text{air}}}{V_{\text{air, steh}}} \quad (11)
\]
where \(V_{\text{air}}\) is the volume of the air which goes into the combustion chamber and \(V_{\text{air, steh}}\) is the stehiometrically required volume of the air necessary for complete combustion of 1 kg fuel. The combustion plant is working with air deficiency when \(\lambda < 1\), and with air excess when \(\lambda > 1\). From techno-economic viewpoint, the losses of the combustion can be reduced in two ways: 1) by reducing the quantity of the unburned fuel and 2) by reducing the quantity of the flue gases, i.e. of the heat losses. This leads to the optimal value \(\lambda_{\text{opt}}\) of the air factor [4]. From environmental viewpoint, it is desired to minimize the quantity of the harmful emissions and the corresponding optimal value of the air factor is \(\lambda_{\text{opt}}\) [4].

By taking into account both the techno-economic and the environmental aspects of combustion operation, it follows that the value \(\lambda\) of the air factor should be kept within the interval \([\lambda_{\text{min}}, \lambda_{\text{opt}}]\). It has been also shown in the praxis that the optimal air factor \(\lambda_{\text{opt}}\) depends on the load factor \(\beta\) defined as:
\[
\beta = \frac{\Phi_{\text{fuel}}}{\Phi_{\text{fuel, max}}} \quad (12)
\]
where \(\Phi_{\text{fuel}}\) and \(\Phi_{\text{fuel, max}}\) are respectively the current and the maximal allowed fuel flowrate. The relation \(\lambda_{\text{opt}} = f(\beta)\) is shown in Fig. 1.

Therefore, the goal is to apply control algorithms that will maintain the air factor on its optimal value with every change of the load factor.

#### Stochastic predictive control of the combustion plant
Here, a reference tracking GP-NMPC control problem for the combustion plant is solved based on its Gaussian process model obtained in previous section:
\[
x_{\text{opt}}(t+1), x_{\text{opt}}(t), \Phi_{\text{opt}}(t), \phi(t) 
\sim \mathcal{N}(\mu(x_{\text{opt}}(t+1)), \sigma^2(x_{\text{opt}}(t+1)))
\] (13)
The control input is \(\phi\) (the angle of the damper for the air flow), the state variable is \(x_{\text{opt}}\) (the percentage of O\(_2\) in the flue gases), and the reference signal is \(r_{\text{ref}}\) (the required percentage of O\(_2\) in the flue gases). The details of the reference generator \(r_{\text{ref}} = f(\Phi_{\text{ref}})\) can be found in [4].

The goal of the GP-NMPC controller is to bring the air factor (respectively the concentration of oxygen in the flue gas) on its optimal value with every change of the load factor and thus to achieve an optimal operation of the combustion plant. Thus, a reference tracking GP-NMPC problem has to be solved, where the task is to have the state variable \(x_{\text{opt}}\) track the reference signal \(r_{\text{ref}}\). For the current \(x_{\text{opt}}(t)\), the reference tracking GP-NMPC solves the following optimization problem:
\[
V'(x_{\text{opt}}(t), r_{\text{ref}}(t), \phi(t-1)) = \min J(U; x_{\text{opt}}(t), r_{\text{ref}}(t), \phi(t-1)) \quad (14)
\]
such that \(x_{\text{opt}, \phi} = x_{\text{opt}}(t)\) and:
\[
30 \leq \phi_{\text{ref}} \leq 60, \quad k = 0, 1, ..., N-1
\]
\[
x_{\text{opt}, \phi, \text{ref}}(t) = x_{\text{opt}}(t), \Phi_{\text{ref}}(t), \phi(t-1) \quad (15)
\]
\[
-\mathcal{N}(\mu(x_{\text{opt}, \phi, \text{ref}}(k)), \sigma^2(x_{\text{opt}, \phi, \text{ref}}(k)))
\quad (16)
\]
with \(U = [\phi_1, \phi_2, ..., \phi_k]\) and the cost function given by:
\[
J(U; x_{\text{opt}}(t), r_{\text{ref}}(t), \phi(t-1)) = \left\| \mu(x_{\text{opt}, \phi, \text{ref}}(k)) - r_{\text{ref}}(k) \right\|^2 + \\
\sum_{k=1}^{N-1} \left\| \mu(x_{\text{opt}, \phi, \text{ref}}(k)) - r_{\text{ref}}(k) \right\|^2 + \left\| \mu(x_{\text{opt}, \phi, \text{ref}}(k)) - r_{\text{ref}}(k) \right\|^2 \quad (17)
\]

The sampling time is \(T_s = 3 [s]\). The prediction horizon is \(N = 5\) and the weighting matrices in the cost function (17) are \(Q = 20, \quad R = 1, \quad P = 20\). An input blocking is applied such that the control input is allowed to change only at time instants 1 and 3 of the prediction horizon. This leads to 2 optimization variables in problem P1 and thus reduces the computational complexity. The performance of the GP-NMPC controller was simulated for the following change in the fuel flowrate: \(\Phi_{\text{fuel}}(t) = 0.7 [\text{kg/s}], t \in [0; 50] \), \(\Phi_{\text{fuel}}(t) = 1.0 [\text{kg/s}], t \in [51; 100] \), \(\Phi_{\text{fuel}}(t) = 1.3 [\text{kg/s}], t \in [101; 150] \). The resulting response is depicted in Fig. 2 to Fig. 4.
Fig. 3. The mean value of the state variable predicted with the Gaussian process model. The solid curve is with the GP-NMPC controller and the dashed curve is the set point.

Fig. 4. The 95% confidence interval of the state variable predicted with the Gaussian process model. The solid curve is with the GP-NMPC controller and the dashed curve is the set point.

CONCLUSIONS

In this paper, an on-line optimization approach for stochastic NMPC is applied to the reference tracking control of a combustion plant based on its Gaussian process model. The GP-NMPC controller brings the air factor on its optimal value with every change of the load factor and thus an optimal operation of the combustion plant is achieved.

REFERENCES