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Copula-based decision support system for quality ranking in the manufacturing of electronically commutated motors

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Abstract End-quality assessment of finished products plays an important role in manufacturing systems. In this paper, we address the end-quality of electronically commutated (EC) motors by subjecting each finished product to a short measurement session. Based on the features calculated from these measurements, the end-quality is assessed by introducing a novel copula-based decision support system (DSS). The proposed DSS provides a full ranking of EC motors by integrating expert's preferences and company's quality standards. This approach overcomes the shortcomings of the traditional regression models, such as partial ranking and inconsistent evaluations with the expert's expectations. We demonstrate the effectiveness of the proposed DSS on a test batch of 840 EC motors.

Keywords decision support system · copula-based regression · EC motors · end-quality assessment

1 Introduction

Quality assessment of finished products is usually the final step in a manufacturing line. The end-quality is assessed by aggregating information contained in the extracted feature set according to a set of pre-defined preferences and rules. Such a task can be implemented by applying concepts of decision support systems (DSS). In this paper we propose a solution for an end-quality assessment of electronically commutated (EC) motors using a copula-based DSS method.

A typical structure of an end-quality assessment system is shown in Figure 1. Such a system has two inputs: features, extracted from the performed measurements, and expert's preferences regarding the final quality evaluation. These inputs are then processed through the two main stages: integration of system's features and expert's preferences, and definition of a copula-based DSS for assessment of overall quality and ranking of finished products. Implementation of these steps has to ensure high sensitivity to the variations in the quality of the finished product. Consequently, each segment of the system has to be custom-made for the problem in hand.

The extracted feature set should contain the most useful features for determining the fault condition of the monitored system (Vachtsevanos et al, 2006). In absence of faults, feature values should belong to the set of nominal (admissible) values. Any discrepancy in one or several features is regarded as a presence of fault, hence indicating decrease in the overall quality. In order to meet such requirements, two steps should be performed: the most informative features should be selected based on existing fault models, and extraction of their values should be performed using fast and accurate signal processing techniques.

The problem of specifying the most informative feature set in the case of electrical motors has been addressed by many authors (Didier et al, 2007; Juričić et al, 2001; Röpke and Filbert, 1994; Sasi et al, 2001; Boškoski et al, 2011). Generally features are extracted from vibration and/or electrical signals. In our particular case, the features were extracted using solely vibration signals. From a plethora of available signal processing methods, we opted for the well established approach using envelope analysis (Peng and Chu, 2004; Jardine et al, 2006; Sawalhi et al, 2007; Randall and Antoni, 2011). As a pre-processing step we used spectral kurtosis (Antoni, 2006) and cyclostationary analysis (Boškoski et al, 2010), due to their capabilities of selecting the most appropriate frequency band for envelope anal-

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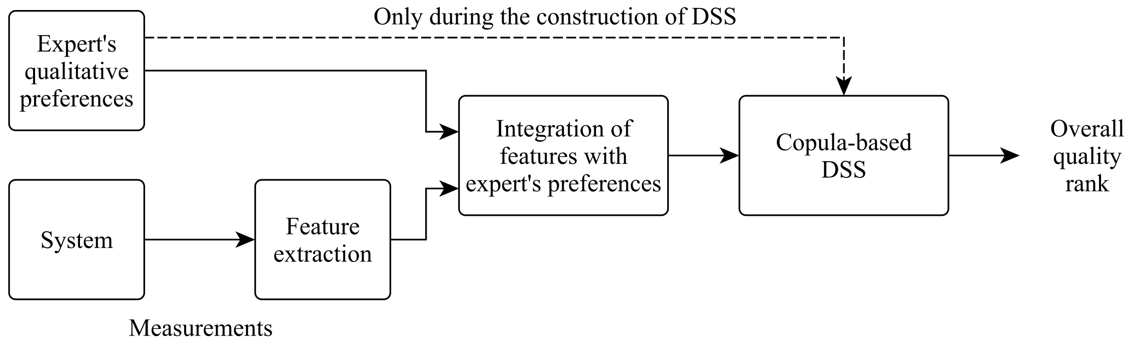

Fig. 1: Structure of the end-quality assessment system

ysis, hence significantly improving the sensitivity of the approach.

The construction of the copula-based DSS starts with integration of features with expert's preferences. The integration addresses the issues of fusing information from the extracted features into an abstract quality rank based on a set of pre-defined expert's preferences. Usually these preferences are expressed using qualitative grades. For this purpose, qualitative aggregation functions are suitable candidates, such as the ones proposed in the Decision Expert (DEX) methodology (Bohanec and Rajkovič, 1990). Within DEX, the expert's preferences are structured into tabular forms and the goal is to determine an aggregation function which most accurately describes the link between the input attributes and the aggregated output. Such an approach has been used in many fields such as environment (Bohanec et al, 2008), agriculture (Pavlovič et al, 2011), agronomy (Žnidaršič et al, 2008), education (Gasar et al, 2003), and health care (Bohanec et al, 2000).

As DEX uses qualitatively described aggregation functions, it leads to partial ranking of the options at hand. To achieve a full ranking of options, we propose an extension of the DEX methodology based on copula functions. Copulas are functions that define the connection between the marginal distributions of the random variables and their joint distribution (Nelsen, 2006). Copulas were successfully applied in petroleum industry (Al-Harthy et al, 2007), finance (Fischer et al, 2009; Bouyé et al, 2000), hydrology (Genest and Favre, 2007), biology (Kim et al, 2008), change detection in images (Mercier et al, 2008) and machine learning (Jaimungal and Ng, 2009). Within the DEX methodology, copula-based regression is employed in order to describe the connection between the input and output attributes. Unlike linear regression functions, which tend to provide partial ranking when two or more attributes receive the same weight value (Mileva-Boshkoska and Bohanec, 2011) or evidential reasoning method which sometimes leads to evaluations that are not in line with the expert's expectations (Boškoski et al, 2011), the usage of DEX and copula-based regression leads to high sensitivity to small variations of the input values. This process produces twofold output information. Firstly, the constructed copula-based DSS yields a grade (also called class) to which the examined EC motor belongs. Secondly, it produces a rank value that can be employed for ordering the EC motors within each grade. Therefore, one can easily specify the position of each finished EC motor within the population of produced units based on its quality rank.

Due to the diverse background of the problem, each segment of the proposed copula-based end-quality assessment system is throughly described. Details about the physical background of the system along with concepts for feature extraction are presented in section 2. The design of the copula-based DSS is presented in section 3. Section 4 describes the actual implementation and section 5 presents the results of the evaluation of the proposed assessment system.

2 Feature extraction

Proper selection of the feature set is crucial for the overall effectiveness of the end-quality assessment. When analysing vibration signals generated under constant operating conditions, feature values are usually the amplitudes of particular spectral components. In the context of EC motors the most frequent mechanical faults are rotor and bearing faults. Therefore, the proposed feature set contains features capable of describing these two groups of faults.

2.1 Rotor faults

Due to improper manufacturing or improper assembly, rotor faults include:

– mass unbalance, and
– misalignment faults.

The presence of either of the faults influences the mass displacement on the rotor, hence changing its moment of inertia. Under constant rotational speed, such a change can be detected by analysing the generated vibrations and it is generally expressed as an increase of the amplitudes of the spectral components at the rotational frequency f_{rot} and its higher harmonics $n \times f_{rot},\ n \in \{2,3,\cdots\}$ (Xu and Marangoni, 1994).

2.2 Bearing faults

Bearings in EC motors are the most susceptible element to mechanical faults. During the manufacturing process the most common causes for introducing bearing faults are improper bearing lubrication, improper mounting and alignment, as well as improper handling during the assembly process.

The detection of these faults is a challenging task. Vibrations, caused by a bearing fault, originate from impacts produced by the rolling elements hitting a damaged place. Each time a hit occurs, an excitation of system eigenmodes occurs in terms of an impulse response $s(t)$. The frequency of occurrence of these impulse responses can be estimated using the rotational speed f_{rot} of the rotating ring and the physical characteristics of the bearing, i.e. the pitch diameter D, the rolling element diameter d, the number of rolling elements Z, and the contact angle α (see Figure 2). Using these parameters the bearing fault frequencies can be calculated according to the relations shown in Table 1 (Tandon and Choudhury, 1999).

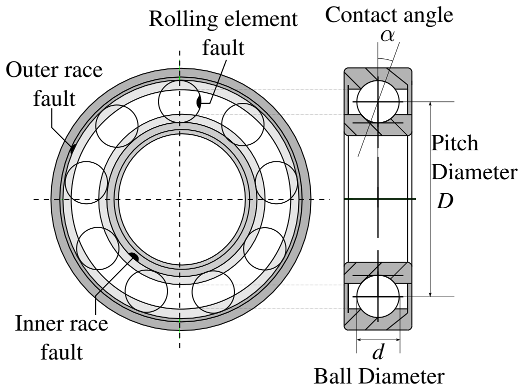

Fig. 2: Bearing dimensions used for the calculation of the bearing's characteristic frequencies

2.3 Complete feature extraction process

The EC motors, used in this research, are all equipped with bearings FAG 6205. The characteristic bearing frequencies listed in Table 1, read as follows:

- $f_{BPFI} = 5.415 f_{rot}$
- $f_{BPFO} = 3.585 f_{rot}$
- $f_{FTF} = 0.398 f_{rot}$
- $f_{BSF} = 2.375 f_{rot}$

Having defined the required feature set, the feature extraction module was built according to the structure shown in Figure 3. The procedure for rotor faults starts by low-pass filtering of the acquired vibration signal. Based on that, three features are extracted: spectral components at f_{rot} and $2 \times f_{rot}$, and the signal variance. Amplitudes of both spectral components are extracted by calculating the RMS value of vibration signal filtered by band-pass filters centred at f_{rot} and $2 \times f_{rot}$ respectively.

Similar procedure was applied for bearing faults. First the vibration signal is band-pass filtered with central frequency f_c located near the excited eigenfrequency. The features, describing bearing faults, are extracted by filtering the envelope signal by means of a set of narrow band-pass filters. Each filter is centred at the corresponding bearing fault frequency.

3 Copula-based decision support system

DEX is a qualitative multi-criteria decision making method that aggregates qualitative multi-attribute options into several qualitative classes. In DEX, the complex decision problem at hand is decomposed into smaller and easily understandable decision components, which are assembled into a hierarchical model. There are two types of attributes in DEX: basic attributes and aggregated ones. The former are the directly measurable attributes. The latter are obtained by aggregating the basic and/or other aggregated attributes. The aggregation process in DEX results into a partial ranking of options, i.e. options that belong to the same class are indistinguishable. However, our goal is to obtain a full ranking of options, i.e. to distinguish among all options in a class.

To obtain a full ranking of options, three steps should be performed, as shown in Figure 4. First, the values of the qualitative attributes $QA_1, \ldots, QA_n$ are mapped into discrete quantitative ones $A_1, \ldots, A_n \in \mathbb{Z}$ (step 1a in Figure 4). The mapping function must preserve the preference order, i.e. the higher the preference of QA_i the greater the value of A_i. In the second step, we estimate regression function[1] $g : \mathbb{R}^n \to \mathbb{R}$ such that

$$A_{agg} = g(A_1, \ldots, A_n), \tag{1}$$

that defines the relation between the aggregated (dependent) attribute A_{agg} and input attributes A_i. The final step ensures consistency between the qualitative and quantitative models. It means that if an option belongs to a quantitative class c, then the output regression value must be in the interval $c \pm 0.5$. To achieve this, for the regression function (1), we

[1] Desppite $A_i \in \mathbb{Z}$, the function $g(A_1, \ldots, A_n)$ is defined in $\mathbb{R}^n$

Table 1: Bearing frequencies (Tandon and Choudhury, 1999)

Name	Relation to the rotational frequency f_{rot}
Bearing pass frequency inner race (BPFI)	$f_{BPFI} = \frac{Zf_{rot}}{2}\left(1+\frac{d}{D}cos\alpha\right)$
Bearing pass frequency outer race (BPFO)	$f_{BPFO} = \frac{Zf_{rot}}{2}\left(1-\frac{d}{D}cos\alpha\right)$
Fundamental train frequency (FTF)	$f_{FTF} = \frac{f_{rot}}{2}\left(1-\frac{d}{D}cos\alpha\right)$
Ball spin frequency (BSF)	$f_{BSF} = \frac{Df_{rot}}{2d}\left(1-\left(\frac{d}{D}cos\alpha\right)^2\right)$

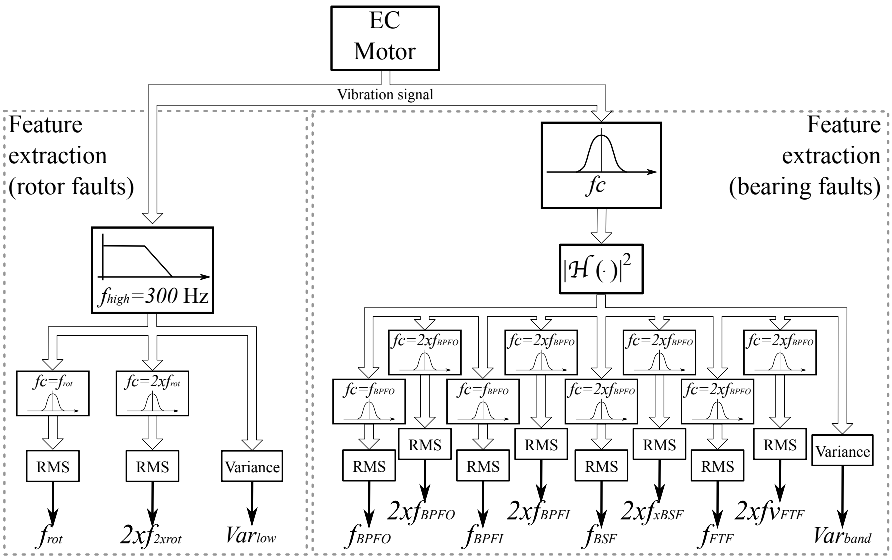

Fig. 3: Structure of the feature extraction module

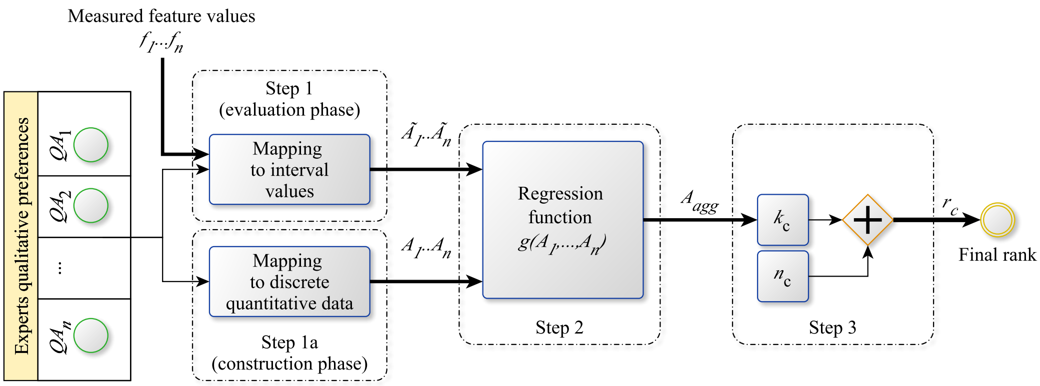

Fig. 4: From qualitative attributes and quantitative featrues to final quantitative evaluation

define set of functions r_c that ensures compliance with the original class c as:

$$r_c(A_1,\ldots,A_n) = k_c g(A_1,\ldots,A_n) + n_c, \quad (2)$$

where k_c and n_c are calculated as:

$$k_c = \frac{1}{max_c - min_c} \quad (3)$$

$$n_c = c + 0.5 - k_c min_c, \quad (4)$$

and max_c and min_c are the maximum and minimum value of the function $g(A_1,\ldots,A_n)$ for a class c. As soon as the model is built, the features may be evaluated. For evaluation, feature values $f_1,..f_n$ are mapped into interval values $c \pm 0.5$ (step 1 in Figure 4) which are then propagated to the regression functions (1) and (2) for final evaluation.

Table 2: Different Archimedean copulas

	$C_\theta(u,v)$	$\varphi_\theta(t)$	$Solve(\frac{\partial C_\theta(u,v)}{\partial u} = q, v)$
Clayton	$\left[\max\left(u^{-\theta} + v^{-\theta} - 1, 0\right)\right]^{-1/\theta}$	$\frac{1}{\theta}\left(t^{-\theta} - 1\right)$	$(1 - u^{-\theta} + (qu^{1+\theta})^{-\frac{\theta}{1+\theta}})^{-\frac{1}{\theta}}$
Frank	$-\frac{1}{\theta}\ln\left(1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1}\right)$	$-\ln\frac{e^{-\theta t} - 1}{e^{-\theta} - 1}$	$\frac{1}{\theta}\log\frac{-e^{\theta}(1-q+qe^{\theta u})}{-e^{\theta}+q-e^{\theta}-q-e^{\theta}u}$
Gumbel-Hougaard	$\exp\left(-\left[(\ln u)^{\theta} + (-\ln v)^{\theta}\right]^{1/\theta}\right)$	$\ln\frac{1-\theta(1-t)}{t}$	*only numerical solution*

3.1 Copula functions

To define the regression function (1), we use a copula-based approach. When using copulas, attributes A_i are regarded as a random variables, for which we can estimate their marginal distributions $F_i(A_i)$. For determining the dependences among the random variables, we need to find their joint distribution $F(A_1,\dots,A_n,A_{agg})$. In order to estimate the multivariate joint densities with a constant estimation accuracy, the required sample size rapidly increases with the number of dimensions (Silverman, 1986). As we deal with small sample sizes, we adopt the copula approach for estimation of the joint density and distribution.

Let the two random variables X_1 and X_2 have marginal distributions $F_1(X_1)$ and $F_2(X_2)$ respectively and joint distribution $F(X_1,X_2)$. According to Sklar's Theorem (Sklar, 1996; Nelsen, 2006), there exists a multivariate distribution function $C_\theta(u,v)$, where θ is a parameter that has to be estimated, and u and v are uniformly distributed random variables on the unit interval $[0,1]$, such that:

$$F(X_1,X_2) = C_\theta(F_1(X_1),F_2(X_2)) = C_\theta(u,v), \tag{5}$$

where

$$u = F_1(X_1),\ u \sim \mathscr{U}(0,1), \\ v = F_2(X_2),\ v \sim \mathscr{U}(0,1). \tag{6}$$

From (5) it follows that copula $C_\theta(u,v)$ is a function that couples the marginal distributions $F_1(X_1)$, $F_2(X_2)$ of the random variable $X = (X_1,X_2)$ with its joint distribution $F(X_1,X_2)$.

From the available methods for copula construction we selected the Archimedean family, where copulas are constructed by using the following relation (Joe, 1997):

$$C_\theta(u,v) = \varphi_\theta^{[-1]}(\varphi_\theta(u) + \varphi_\theta(v)) \tag{7}$$

where $\varphi_\theta(\cdot)$ is called a generator function and $\varphi_\theta^{[-1]}(\cdot)$ is

$$\varphi_\theta^{[-1]}(t) = \begin{cases} \varphi_\theta^{-1}(t), & \text{if } 0 \le t \le \varphi_\theta(0); \\ 0, & \text{if } \varphi_\theta(0) \le t \le \infty. \end{cases} \tag{8}$$

The generator function $\varphi_\theta(t) : [0,1] \to [0,\infty]$ must be continuous and strictly decreasing. Table 2 shows different types of Archimedean copulas constructed with the different generators $\varphi_\theta(t)$ (Nelsen, 2006).

3.2 Fully nested Archimedean copulas

Equation (5) couples only two marginal distributions. In order to deal with multivariate random variables, we need to extend the bivariate copula into a multivariate one. In this paper we use the bivariate copula as a building block for obtaining a fully nested Archimedean copula (FNAC) (Hofert, 2010; Berg and Aas, 2009), such as the one given in Figure 5. FNAC is a tree like structure which is obtained using an iterative procedure that starts with coupling two random variables. For example, in Figure 5 we couple u_1 and u_2 into copula $C_{\theta_1}(u_1,u_2)$ with parameter θ_1. In all subsequent iterations, the obtained copula is coupled with a new random variable, for example copula C_{θ_1} is coupled with u_3 into $C_{\theta_2}(C_{\theta_1},u_2)$ with parameter θ_2, and so on. The final output of the topmost copula reads:

$$C_{\theta_4}(u_1,u_2,u_3,u_4,u_5) = \\ C_{\theta_4}\left(u_5, C_{\theta_3}\left(u_4, C_{\theta_2}\left(u_3, C_{\theta_1}\left(u_1,u_2\right)\right)\right)\right). \tag{9}$$

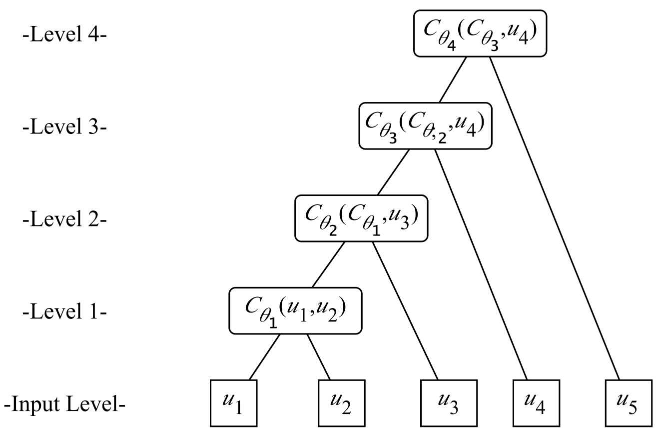

Fig. 5: Fully nested Archimedean copula

In general case, FNAC structure with n input variables has $n-1$ parameters θ. The final function (9) represents a valid copula only if the following condition is fulfilled (Rachev, 2003):

$$\theta_1 > \theta_2 > \dots > \theta_{n-1} \tag{10}$$

where θ_1 is the parameter of the most nested copula, θ_2 is the parameter of the second most nested copula and so on. The estimations of the values of $\theta_i,\ i = 1,\ldots,n-1$, are obtained using the maximum likelihood algorithm (Brent, 1993; Forsythe et al, 1976).

Sometimes, it is not possible to build a FNAC that satisfies condition (10) for the desired order of variables in the FNAC. In these cases we look for a valid FNAC in the set of all FNACs obtained by permuting the order of the variables entering the FNAC.

3.3 Regression using FNAC

For bivariate copula, a suitable regression function linking the two variables can be found by Algorithm 1 (Nelsen, 2006).

Algorithm 1 Regression using Frank bivariate copula

```
∂Cθ(u,v)/∂u = q                       ▷ calculate median regression for q = 1/2
v ← Solve(∂Cθ(u,v)/∂u = q, v)         ▷ see Table 2 for different copulas
u ← F1(x1)                            ▷ replace u by F1(x1)
v ← F2(x2)                            ▷ replace v by F2(x2)
```

In Algorithm 1, we first differentiate the copula function over the input variable to get the quantile regression function. Then we set the obtained expression equal to a quantile value q. In case when $q = 0.5$, we perform median regression. The advantage of using median regression as a measure of centre is due to its ability to resist the strong effect of outliers (Walters et al, 2006). The median regression curve v based on the different Archimedean copulas is given in Table 2. Here we work with Frank copula, for which the median regression curve v is:

$$v = \frac{1}{\theta}\log\frac{-e^{\theta}(1-q+qe^{\theta u})}{-e^{\theta}+q-e^{\theta}-q-e^{\theta}u}. \tag{11}$$

Finally, by replacing u with $F_1(X_1)$ and v with $F_2(X_2)$ in (11), the final copula-based regression function reads:

$$g = F_2^{-1}(x_2). \tag{12}$$

To use Algorithm 1, the dependent variable should be placed in the right most position in the FNAC, such as the variable u_5 in Figure 5. Algorithm 2, on the other hand, gives a general solution for copula-based regression with FNAC for n random variables where the regression variable seats in an arbitrary position p (Boshkoska and Bohanec, 2012).

Algorithm 2 performs regression in iterations. It starts with regression at the topmost copula. The obtained regression values are propagated downwards in the hierarchical structure, where the value of q is replaced with the regression values of v. The iterations continue until the dependent

Algorithm 2 Regression algorithm for FNAC structure and dependent variable in the p position

```
i ← n+1                         ▷ n is the number of random variables/attributes
q ← 0.5                         ▷ calculate median regression for q = 1/2
v ← Solve(∂Cθ(u,v)/∂u = q, v)   ▷ calculate v; see Table 2 for different copulas
repeat:
    q ← v                       ▷ replace q with the value v
    v ← Solve(∂Cθ(u,v)/∂u = q, v)   ▷ recalculate the new value of v
    i ← i − 1
until i = p   ▷ p is the regression variable position in input level in Figure 5
call Algorithm 1, for q = v
```

variable p in FNAC is reached. Finally, the regression function is obtained as in Algorithm 1, for $q = v$ from the last iteration. The same analysis can be used for other two types of copulas given in Table 2.

4 Implementation within the quality assessment system

The implementation of the copula-based quality assessment system follows the steps shown in Figure 4. Firstly, one has to transform the decision maker's preferences into a qualitative tabular functions. Then, a mapping from qualitative attributes into quantitative ones has to be performed. Finally, for each quantitative table function, a copula function is defined. After the completion of these steps, one can apply the algorithms 1 and 2 for copula-based regression described in the previous section.

4.1 DEX hierarchical model

Assessing the overall motor quality rank directly from the measured features is a rather difficult task. Therefore, the problem is transformed into a hierarchical decision making model in which the overall *mechanical quality* rank is obtained by aggregating two simpler attributes: *rotor quality* and *bearings quality*. The former attribute can be directly assessed from the measured features described in section 2.1. The latter attribute is still complex as it can be further decomposed into four simpler attributes: *inner ring quality*, *outer ring quality*, *quality of the rolling elements* and *quality of the bearing cage*. These four attributes can be assessed from the measured features describing bearing condition, as shown in section 2.2. Based on this logical structure we built a DEX hierarchical model, which is shown in the first column of Table 3, where the aggregated attributes are given in bold upper cases.

Following the expert's preferences and knowledge, each attribute in the proposed hierarchical structure was described or aggregated using the expert's defined scale with

Table 3: DEX model tree and qualitative and quantitative evaluations of EC motors 744 and 9

Attribute	**Evaluation of motor 744**		**Evaluation of motor 9**	
	Qualitative	Quantitative	Qualitative	Quantitative
MECHANICAL QUALITY	**not satisfactory**	1.3389	very good	3.3056
ROTOR QUALITY	**top**	4.9702	**top**	5.0302
f_{rot}	**top**	4.7851	**top**	5
$2 \times f_{rot}$	**top**	5	**top**	5
Variance	**top**	5	**top**	5
BEARINGS QUALITY	**not satisfactory**	0.7592	good	1.7899
INNER RING	good	1.9583	very good	2.8051
BPFI	good	1.7096	very good	2.8227
2×BPFI	very good	2.6319	excellent	2.8830
OUTER RING	good	1.8009	good	2.1889
BPFO	good	1.9043	very good	3.0203
2×BPFO	very good	2.1660	good	2.0238
ROLL ELEMENTS	**not satisfactory**	0.8810	very good	2.8454
BSF	**not satisfactory**	1.2396	very good	2.9361
2×BSF	very good	2.6100	excellent	2.9288
FTF	good	2.6279	very good	3.0974

five qualitative values:

$$Q_C = \{not\ satisfactory, good, very\ good, excellent, top\}.$$

For instance, the aggregation of the basic attributes *BPFI* and 2×*BPFI* into attribute *Inner ring* is given in the first three columns of Table 4. These aggregations may be interpreted as a set of **if-then** rules, for instance, the last row in Table 4 can be interpreted as follows:

if *BPFI* is *Top*
and $2 \times BPFI$ is *Top*
then *Inner ring* is *Top*.

4.2 Qualitative to quantitative value mapping

To obtain the quantitative model, we map each of the qualitatively defined expert rules and preference values into a quantitative one. The preferences of the qualitative values are given as:

$$not\ satisf. \prec good \prec very\ good \prec excellent \prec top \quad (13)$$

where the sign $\prec$ stands for "is strictly less preferred than". Consequently, these values are mapped into $\{1, 2, 3, 4, 5\}$ respectively. The sign $\prec$ is mapped into <, where < stands for 'is greater than'. The mapping ensures that the more preferred values are mapped into greater numbers. An example of the mapping is given in the last three columns of Table 4, where the qualitative values of attributes *BPFI*, 2× *BPFI* and *Inner ring* are mapped into quantitative values of X_1, X_2 and Y respectively.

Table 4: Expert defined rules for aggregation of the attribute *Inner ring* and mapping from the qualitative attribute values into quantitative ones.

BPFI	**2xBPFI**	**Inner ring**	X_1	X_2	Y
'not satisf.'	*'not satisf.'*	*'not satisf.'*	1	1	1
'not satisf.'	*'good'*	*'not satisf.'*	2	1	1
'not satisf.'	*'very good'*	*'not satisf.'*	1	3	1
'not satisf.'	*'excellent'*	*'not satisf.'*	1	4	1
'not satisf.'	*'top'*	*'not satisf.'*	1	5	1
'good'	*'not satisf.'*	*'not satisf.'*	2	1	1
'good'	*'good'*	*'good'*	2	2	2
'good'	*'very good'*	*'good'*	2	3	2
'good'	*'excellent'*	*'very good'*	2	4	3
'good'	*'top'*	*'very good'*	2	5	3
'very good'	*'not satisf.'*	*'not satisf.'*	3	1	1
'very good'	*'good'*	*'good'*	3	2	2
'very good'	*'very good'*	*'very good'*	3	3	3
'very good'	*'excellent'*	*'very good'*	3	4	3
'very good'	*'top'*	*'excellent'*	3	5	4
'excellent'	*'not satisf.'*	*'not satisf.'*	4	1	1
'excellent'	*'good'*	*'very good'*	4	2	3
'excellent'	*'very good'*	*'very good'*	4	3	3
'excellent'	*'excellent'*	*'excellent'*	4	4	4
'excellent'	*'top'*	*'excellent'*	4	5	4
'top'	*'not satisf.'*	*'not satisf.'*	5	1	1
'top'	*'good'*	*'very good'*	5	2	3
'top'	*'very good'*	*'excellent'*	5	3	4
'top'	*'excellent'*	*'excellent'*	5	4	4
'top'	*'top'*	*'top'*	5	5	5

4.3 Integration of the feature values and the expert's preferences

The integration between the measured feature values and the expert's preferences is performed using fuzzification.

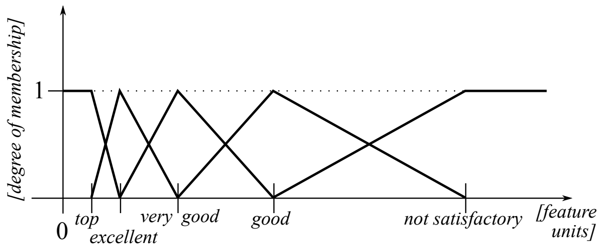

Fig. 6: Intervals for mapping feature values to quantitative ones

The expert's preference is towards motors with lower vibrations, hence lower feature values are more preferred. The maximum allowed value for each feature determines the limit for the *not satisfactory* grade. This limit was determined either by governing standard rules or by the company's quality requirements. The remaining interval below the limit for the *not satisfactory* grade was divided dyadically, as shown in Figure 6. Such mapping ensures more sensitivity at lower feature values. The feature values f_i are fuzzified and mapped accordingly into the expert's defined interval $[1,5]$, employing the relation:

$$\tilde{A}_i(f_i) = \sum_n \mu_n(f_i) Q_n. \quad (14)$$

Here $\mu_n(f_i)$ is the membership function of the n^{th} rule as given in Figure 6, and $Q_n \in \{1,2,3,4,5\}$ is the class value.

4.4 Constructing the copula-based regression functions

According to the model shown in Table 3 there are six aggregation tables. For each table, a FNAC based on the Frank bivariate copula was built. Hence six copula-based regression functions were derived. The obtained values from (14) enter the appropriate copula-based regression functions, and for each of them a copula-based regression value is calculated. Afterwards, the calculated value is normalised in order to retain consistency with the qualitative model as defined in (2)-(4) (Stage 3 in Figure 4). The obtained values are propagated in the higher hierarchical level, where they are used as inputs in the next regression function. The procedure is recursively repeated up to the topmost table. The result of the topmost table is regarded as the overall quality rank for each motor.

5 Results

5.1 The assessment rig

Each EC motor is tested using the assessment rig shown in Figure 7. The rig consists of a fixed pedestal on top of which a metal disk is positioned. The metal disk holds three rubber dampers that suspend the tested EC motor. The experiment starts by positioning the EC motor vertically on the rubber dampers in such a way that the drive-end bearing is on the bottom. Afterwards, two accelerometers are positioned on the motor housing nearest to the both bearings. The test-rig minimises the environmental influence, hence guaranteeing sufficiently constant experimental conditions.

The data acquisition process commences as soon as the nominal rotational speed is reached. Firstly, both vibration signals are low-pass filtered with cut-off frequency at 22 kHz. Afterwards, both signals are sampled at 60 kHz. During the whole data acquisition process the nominal rotational speed of $f_{rot} = 38$ Hz is maintained. Each acquisition process lasts 8 seconds. After finishing the acquisition the motor is decelerated down to the stop position.

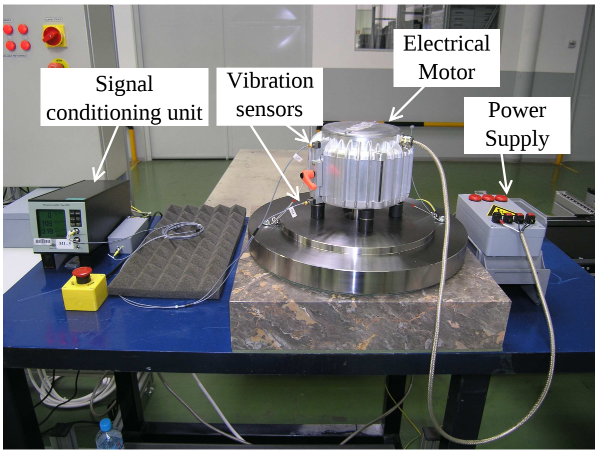

Fig. 7: The prototype assessment point

5.2 Results on a test batch of motors

During the evaluation process we analysed a test batch of 840 EC motors. The overall quality rank is shown in Figure 8. We have to note that for the purpose of testing the system on small differences in data, during the initial start-up of the line, we have intentionally introduced motors with various mechanical faults. Consequently, there are many motors with different overall quality rank.

From the results shown in Figure 8, it is clearly visible that the quality ranks of the tested motors are spread over the interval $[0.5, 5.5]$. This is an indication that the proposed copula-based DSS is highly sensitive even to minor variations in the motor quality. Unlike methods that use weighted utility functions, where options with *not satisfactory* feature values are ranked highly, this approach averts such performance. Such example is given in Boškoski et al (2011),

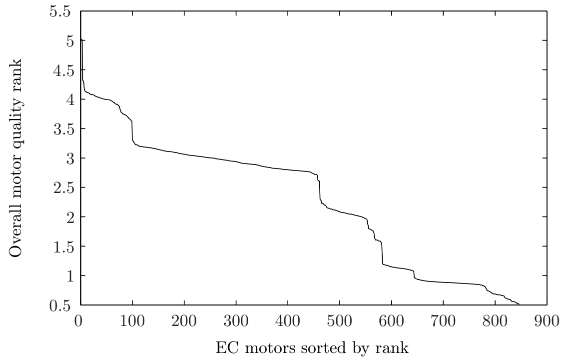

Fig. 8: Rankings obtained with Frank FNAC

where the first 130 motors were evaluated using evidential reasoning approach which lead to cases where final evaluations were inconsistent with the expert's preferences.

Besides the overall quantitative evaluation, the calculated rank also shows the class of each EC motor.

5.3 Detailed analysis of two characteristic examples

The effectiveness of the proposed copula-based DSS can be best illustrated by detailed analysis of two characteristic cases. One case refers to the dominance of the *not satisfactory* grade and the other on ranking of motors whose quality belongs between two adjacent grades.

The first case From the expert's preferences (see Table 4) it is clearly visible that in case that any of the attributes acquires a value that belongs to the grade *not satisfactory*, the examined motor will obtain overall qualitative rank that belongs to the lowest class. The 744$^{\text{th}}$ EC motor is the case with the highest quality rank from the class of *not satisfactory* motors. The measured features for this motor are given in Table 3.

From Table 3 it is visible that the rotor quality belongs to the highest grade, denoted as *top*. Namely, the values of f_{rot}, $2 \times f_{rot}$ and *Variance* belong to the interval $[5 \pm 0.5]$, where the first value denotes the class *top*. Additionally, one may notice that bearing features mostly belong to the qualitative class *good*, for instance FTF, $BPFO$ and $BPFI$ which have values in the interval $[2 \pm 0.5]$, and *very good*, such as $2 \times BPFO$, $2 \times BPFI$ and $2 \times BSF$ that have values in the interval $[3 \pm 0.5]$. Unlike them one feature describing the condition of the rolling element *BSF* has a value that belongs to the interval $[1 \pm 0.5]$, which is a quantitative employment of the *not satisfactory* class. This qualitative value is propagated through all levels of the hiearhical structure of the model, hence leading to *not satisfactory* evaluation of the higher level aggregated attributes *Roll elements, Bearing quality* and final qualitative evaluation of *Mechanical quality*. Consequently, the overall quality rank is just 1.333, which clearly states that the particular EC motor is of *not satisfactory* quality.

The second case The second case is the 9$^{\text{th}}$ EC motor, whose overall quality rank is 3.3056, which belongs to the qualitative class *very good*. The calculated features for this EC motor are given in the last column in Table 3.

According to the measured features, the bearing quality of the 9$^{\text{th}}$ motor can be easily graded as *very good*, since most of the features have value from the interval spanned by this grade. Still, the qualitative value of the attribute $2\times$*BPFO* is *good*, and this value is propagated up in the hiearchy leading to qualitative evaluation of *Bearing Quality* to *good*, as defined by the expert's preferences. Rotor features, on the other hand, undoubtedly state that this particular case has *top* quality of the rotor. Consequently, the overall motor quality is *very good*, however, the numerical rank suggests that the quality is very close to the next higher class *excellent*.

The two examples show that the hierarchy of attributes aids the process of integration of expert's preferences into the final quality evaluation of motors.

6 Conclusion

The quality ranking of EC motors was regarded as a hierarchical decision making task, in which the final motor's quality is aggregated from the quality of its components. The proposed solution is a copula-based decision support system. The input of the system is a set of measured features calculated from the acquired vibrations generated by the examined EC motor. Furthermore, the system employes available expert's knowledge condensed in DEX qualitative tabular form. Employing copula-based regression functions resulted into a full quality ranking of EC motors. The system was evaluated on a batch of 840 motors.

The merging of expert's knowledge with DEX, and employment of copula-based regression leads to a final evaluation system with four properties. First, the qualitative evaluation of each EC motor provides easily understandable quality description. Second, the system has ability of distinguishing small variations of the input features. Therefore, each EC motors is assigned with a quantitative value, leading to distinct evaluation of all EC motors in the test batch. Third, the hierarchical decomposition of the problem gives explanation how the qualities of each of the lower level components lead to the final evaluation. Therefore, besides the process of quality assessment, such a system can be seamlessly employed as a fault detection module that is able of performing fault evaluation too. Finally, the proposed evaluation system for EC motors leads to rankings that are fully in compliance with the decision maker's (or expert's) preferences and the required regulations.

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