B.5. Session
MODEL PREDICTIVE CONTROL
(*MPC*)

EXPLICIT STOCHASTIC MODEL PREDICTIVE CONTROL OF GAS-LIQUID SEPARATOR BASED ON GAUSSIAN PROCESS MODEL[1]

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Abstract: Recently, an approximate approach to explicit stochastic Nonlinear Model Predictive Control (NMPC) based on a Gaussian process model was proposed. A significant advantage of the Gaussian process models is that they provide information about prediction uncertainties. On the other hand, an explicit solution to the stochastic NMPC problem would allow efficient on-line computations as well as verifiability of the implementation. In this paper, an explicit stochastic NMPC controller for a pilot gas-liquid separation plant is designed, based on a Gaussian process dynamic model.

Key words: Explicit Stochastic Model Predictive Control, Gaussian Process Models, Multi-parametric Nonlinear Programming.

INTRODUCTION

Nonlinear Model Predictive Control (NMPC) involves the solution at each sampling instant of a finite horizon optimal control problem subject to nonlinear system dynamics and state and input constraints [1]. Several approaches to *explicit* solution of NMPC problems have been suggested in the literature [2], [3], [4], [5]. The benefits of an *explicit* solution, in addition to the efficient on-line computations, include also verifiability of the implementation. In [2], [3], [4], approaches for off-line computation of *explicit* sub-optimal piecewise linear (PWL) predictive controllers for general nonlinear systems with state and input constraints have been developed, based on the multi-parametric Nonlinear Programming (mp-NLP) ideas [6]. In [7], a parallel computing algorithm for design of *explicit* NMPC controllers has been proposed, which is a parallel implementation of the approximate mp-NLP approach in [4]. It exploits the multi-core computer architectures available nowadays and leads to a significant decrease of the off-line computational efforts associated to the design of *explicit* MPC controllers.

Mathematical models of engineering systems usually contain some amount of uncertainty (typically unknown additive disturbances and/or uncertain model parameters). In the robust MPC problem formulation, the model uncertainty is taken into account. In some applications, the system to be controlled is described by a stochastic model where the probabilistic distribution of the uncertainty is assumed to be known. Several approaches to *on-line* constrained NMPC based on stochastic models have been proposed in [8], [9], [10]. In [11], an approximate mp-NLP approach to *explicit* solution of feedback stochastic NMPC problems has been developed.

The stochastic NMPC approaches [8]–[11] are based on *parametric* probabilistic models. Alternatively, the stochastic systems can be modeled with *non-parametric* models, which can offer a significant advantage compared to the *parametric* models. This is related to the fact that the *non-parametric* probabilistic models provide information about prediction uncertainties which are difficult to evaluate appropriately with the *parametric* models. The Gaussian process model is an example of a *non-parametric* probabilistic black-box model. Its use and properties for modelling are reviewed in [12]. The use of Gaussian processes in the modelling of dynamic systems is a relatively recent development e.g. [13]. An *on-line* optimization approach and an *explicit* approximate approach to stochastic NMPC based on Gaussian process models have been proposed in [14], [15] and in [16], respectively.

In this paper, an *explicit* stochastic NMPC controller for a pilot gas-liquid separation plant is designed, based on a Gaussian process dynamic model. For this purpose, a parallel implementation of the approach in [16] is applied, which uses the parallel mp-NLP algorithm in [7].

FORMULATION OF STOCHASTIC NMPC PROBLEM

Consider a stochastic system described by an uncertain nonlinear discrete-time model:

$$x(t+1) = f(x(t), u(t)) + \xi(t) \qquad (1)$$

where $x(t) \in \Re^n$ and $u(t) \in \Re^m$ are the state and input variables, $\xi(t) \in \Re^n$ are Gaussian disturbances, and $f : \Re^n \times \Re^m \rightarrow \Re^n$ is a nonlinear continuous function. The uncertainty consists in that the analytical expression of $f(x,u)$ is not known and neither are the mean values and the covariances of the disturbances $\xi(t)$. With Gaussian process modelling, the relationship (1) is represented in the form:

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$$y(t) = f_G(z(t)) + \xi(t) \quad (2)$$

where $y(t) = x(t+1) \in \Re^n$ is the model output and $z(t) = [x(t),\, u(t)] \in \Re^{n+m}$ is the model input. Suppose that we have an output data set $Y_i = [y_i(0), y_i(1), ..., y_i(M-1)]$, $i = 1, 2, ..., n$ corresponding to an input data set $\mathbf{Z} = [z(0), z(1), ..., z(M-1)]$. Assume that the relationship (2) is approximated with Gaussian processes with distributions:

$$Y_1 \sim \mathrm{N}\ (0, \boldsymbol{\Sigma}_1)\ ,\ Y_2 \sim \mathrm{N}\ (0, \boldsymbol{\Sigma}_2)\ ,\ \ldots\ ,\ Y_n \sim \mathrm{N}\ (0, \boldsymbol{\Sigma}_n) \quad (3)$$

where the covariance functions $\Sigma_{1,rq} = \mathrm{Cov}_1(y_1(r), y_1(q)) = C_1(z(r), z(q))$, $\ldots$, $\Sigma_{n,rq} = \mathrm{Cov}_n(y_n(r), y_n(q)) = C_n(z(r), z(q))$ with $r = 0, 1, ..., M-1$, $q = 0, 1, ..., M-1$, depend on the given input and output data sets. Having obtained the Gaussian process model (3), the probability distribution of the output $y(M)$ corresponding to a new input $z(M)$ can be determined as:

$$y(M) | z(M), (\mathbf{Z}, \mathbf{Y}) \sim \mathrm{N}\ (\mu_y(M), \sigma_y^2(M)) \quad (4)$$

Here, $\mu_y(M) = [\mu(y_1(M)), ..., \mu(y_n(M))]$ and $\sigma_y^2(M) = [\sigma^2(y_1(M)), ..., \sigma^2(y_n(M))]$ (with $\mu(y_i(M))$ and $\sigma^2(y_i(M))$ denoting the mean and the variance of the output variable $y_i(M)$, $i = 1, 2, ..., n$), and $\mathbf{Y} = [Y_1, Y_2, ..., Y_n]$. For a multi-step ahead prediction, the following applies:

$$\begin{aligned} &y(M+k) | z(M+k), (\mathbf{Z}, \mathbf{Y}) \sim \mathrm{N}\ (\mu_y(M+k), \sigma_y^2(M+k)) \\ &k \geq 1 \end{aligned} \quad (5)$$

Now, suppose the initial state $x(t) = x_{t|t}$ and the control inputs $u(t+k) = u_{t+k}$, $k = 0, 1, \ldots, N-1$ are given. Then, by taking into account that $y(t) = x(t+1)$ and $z(t) = [x(t),\, u(t)]$, from (5) we obtain the probability distribution of the predicted states $x_{t+k+1|t}$, $k = 0, 1, \ldots, N-1$ which correspond to the given initial state $x_{t|t}$ and control inputs u_{t+k}, $k = 0, 1, \ldots, N-1$:

$$\begin{aligned} &x_{t+k+1|t} \,|\, x_{t+k|t}, u_{t+k} \sim \mathrm{N}\ (\mu(x_{t+k+1|t}), \sigma^2(x_{t+k+1|t})) \\ &k = 0, 1, \ldots, N-1 \end{aligned} \quad (6)$$

The 95% confidence interval of the random variable $x_{t+k+1|t}$ is $[\mu(x_{t+k+1|t}) - 2\sigma(x_{t+k+1|t}); \mu(x_{t+k+1|t}) + 2\sigma(x_{t+k+1|t})]$, where $\sigma(x_{t+k+1|t})$ is the standard deviation.

Here, we consider a regulation problem where the goal is to steer the state vector $x(t)$ to the origin. Suppose that a full measurement of the state $x(t)$ is available at the current time t. For the current $x(t)$, the regulation stochastic NMPC solves the following optimization problem:

Problem P1:

$$V^*(x(t)) = \min_U J(U, x(t)) \quad (7)$$

subject to $x_{t|t} = x(t)$ and:

$$\mu(x_{t+k|t}) - 2\sigma(x_{t+k|t}) \geq x_{\min}\ ,\ k = 1, \ldots, N \quad (8)$$

$$\mu(x_{t+k|t}) + 2\sigma(x_{t+k|t}) \leq x_{\max}\ ,\ k = 1, \ldots, N \quad (9)$$

$$u_{\min} \leq u_{t+k} \leq u_{\max}\ ,\ k = 0, 1, \ldots, N-1 \quad (10)$$

$$\begin{aligned} \max\{ &\left\| \mu(x_{t+N|t}) - 2\sigma(x_{t+N|t}) \right\|, \\ &\left\| \mu(x_{t+N|t}) + 2\sigma(x_{t+N|t}) \right\| \} \leq \delta \end{aligned} \quad (11)$$

$$\begin{aligned} &x_{t+k+1|t} \,|\, x_{t+k|t}, u_{t+k} \sim \mathrm{N}\ (\mu(x_{t+k+1|t}), \sigma^2(x_{t+k+1|t})) \\ &k = 0, 1, \ldots, N-1 \end{aligned} \quad (12)$$

with $U = [u_t, u_{t+1}, ..., u_{t+N-1}]$ and the cost function given by:

$$J(U, x(t)) = \sum_{k=0}^{N-1} \left[\left\| \mu(x_{t+k|t}) \right\|_Q^2 + \left\| u_{t+k} \right\|_R^2 \right] + \left\| \mu(x_{t+N|t}) \right\|_P^2 \quad (13)$$

Here, N is a finite horizon and $P, Q, R \succ 0$ are weighting matrices. It is assumed that $x_{\min} < 0 < x_{\max}$ and $u_{\min} < 0 < u_{\max}$. From a stability point of view it is desirable to choose δ in (11) as small as possible [17]. However, due to the uncertainty of the $x_{t+N|t}$ prediction, characterized by the variance $\sigma^2(x_{t+N|t})$, the feasibility of problem P1 will rely on δ being sufficiently large. A part of the NMPC design will be to address this tradeoff.

The optimization problem P1 can be formulated in a compact form as follows:

Problem P2:

$$V^*(x) = \min_U J(U, x) \quad \text{subject to} \quad G(U, x) \leq 0 \quad (14)$$

The problem P2 defines an mp-NLP, since it is NLP in U parameterized by x. An optimal solution to this problem is denoted $U^* = [u_t^*, u_{t+1}^*, ..., u_{t+N-1}^*]$ and the control input is chosen according to the receding horizon policy $u(t) = u_t^*$. Define the set of N-step feasible initial states as follows:

$$X_f = \{ x \in \Re^n \mid G(U, x) \leq 0 \text{ for some } U \in \Re^{Nm} \} \quad (15)$$

In parametric programming problems one seeks the solution $U^*(x)$ as an *explicit* function of the parameters x in some set $X \subseteq X_f \subseteq \Re^n$ [6].

DESIGN OF EXPLICIT STOCHASTIC NMPC OF GAS-LIQUID SEPARATOR

Gaussian process model of the gas-liquid separator

The gas-liquid separation unit (Fig. 1) is a semi-industrial process plant that forms part of a larger pilot plant. The role of the separation unit is to capture flue gasses under low pressure from effluent channels by means of water flow, to cool them down and then supply them under high-enough pressure to other parts of the pilot plant.

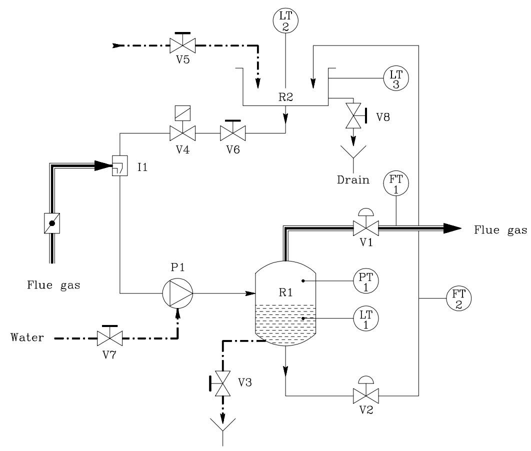

Fig. 1. Process scheme of the separation unit.

The flue gasses coming from the effluent channels are "pooled" by the water flow into the water circulation pipe through the injector I_1. The water flow is generated by the pump P_1 (water ring). The speed of the pump is kept constant. The pump feeds the mixture of water and gas into the separator R_1 where gas is separated from water. Hence, the accumulated gas in R_1 forms a sort of “gas cushion” with increased internal pressure. Owing to this pressure, the flue gas is blown out from R_1 into the next unit of the pilot plant. On the other side the “cushion” forces water to circulate back to the reservoir R_2. The quantity of water in the circuit is constant. If for some reason additional water is needed, the water supply path through the valve V_5 is utilized. More details of the plant can be found in [18].

In [19], a Gaussian process model of the separator’s dynamics has been obtained based on measurement data for the input and the output signals, sampled with sampling time of 20 [s]. The model is composed of two parts: one is the sub-model that predicts the pressure p :

$$p(t+1) = f_1(p(t), u_1(t), h(t)) \quad (16)$$

and the other is the sub-model that predicts the liquid level h :

$$h(t+1) = f_2(h(t), u_2(t), p(t)) \quad (17)$$

Here, u_1 and u_2 are control inputs, which represent the openness of valves V_1 and V_2, respectively. The obtained Gaussian process sub-models have a squared exponential (Gaussian) covariance function:

$$C^j(z^j(r), z^j(q)) = v_1^j \exp\left[-\frac{1}{2}\sum_{i=1}^{D} w_i^j (z_i^j(r) - z_i^j(q))^2\right] + v_0^j \alpha_{rq},\ j = 1,2 \quad (18)$$

where $j = 1$ is associated to model (16), $j = 2$ corresponds to model (17), $D = 3$ is the number of input signals in both models, r and q are discrete time instances, and α_{rq} is the Kronecker operator. Thus, the model (16) has covariance function $C^1(z^1(r), z^1(q))$ (where $z^1 = [p, u_1, h]$) with the following parameters:

$$[w_1^1, w_2^1, w_3^1, v_0^1, v_1^1] = [1.71,\ 0.10,\ 1.68,\ 8.26\times10^{-5},\ 0.27] \quad (19)$$

and the model (17) has covariance function $C^2(z^2(r), z^2(q))$ (where $z^2 = [h, u_2, p]$) with the following parameters:

$$[w_1^2, w_2^2, w_3^2, v_0^2, v_1^2] = [4.81,\ 97.19,\ 848.51,\ 2.55\times10^{-4},\ 7.28] \quad (20)$$

Design and performance of explicit stochastic NMPC

Based on the Gaussian process model (16)-(17), an *explicit* stochastic NMPC controller for the gas-liquid separation plant is designed. For this purpose, a parallel implementation of the approach in [16] is applied, which uses the parallel mp-NLP algorithm in [7]. The following constraints are imposed on the control inputs:

$$0 \le u_1(t) \le 1,\ 0 \le u_2(t) \le 1 \quad (21)$$

The set point values for the pressure and the liquid level are:

$$p^* = 0.5\ [\text{bar}],\ h^* = 1.4\ [\text{m}] \quad (22)$$

The model steady state values of the two control inputs corresponding to these set point values are:

$$u_1^* = 0.47,\ u_2^* = 0.848 \quad (23)$$

The NMPC problem formulation has a slightly different form than the one described in the previous section. Thus, the aim is to minimize the cost function:

$$J(U, x(t)) = \sum_{k=0}^{N-1}\left[\left\|\mu(x_{t+k|t}) - x^*\right\|_Q^2 + \left\|u_{t+k} - u^*\right\|_R^2\right] + \left\|\mu(x_{t+N|t}) - x^*\right\|_P^2 \quad (24)$$

with $x = [p, h]$, $x^* = [p^*, h^*]$, $u = [u_1, u_2]$, $u^* = [u_1^*, u_2^*]$, subject to the inputs constraints (21) and the Gaussian process model (16)-(17). The horizon is $N = 5$ and the weighting matrices are $Q = P = \text{diag}\{1, 200\}$, $R = \text{diag}\{0.5, 0.5\}$.

In [3], a condition on the tolerance of the cost function approximation error has been derived such that the asymptotic stability of the nonlinear system in closed-loop with the approximate *explicit* NMPC is guaranteed. According to this condition, the tolerance is chosen to be dependent on the state, which would lead to a state space partition with less complexity in comparison to that corresponding to an uniform tolerance. Here, a similar approach is applied and the tolerance is chosen to be $\bar{\varepsilon}(X_0) = \max(\bar{\varepsilon}_a, \bar{\varepsilon}_r \min_{x\in X_0} V^*(x))$, where $\bar{\varepsilon}_a = 0.005$ and $\bar{\varepsilon}_r = 0.1$ are the absolute and the relative tolerances. Here, $X_0 \subset X$, where $X = [0, 1]\times[0.4, 1.8]$ is the state space to be partitioned. The state space partition of the *explicit* stochastic NMPC controller is shown in Fig. 2. The partition has 487 regions and 13 levels of search. Totally, 21 arithmetic operations are needed in real-time to compute the control input (13 comparisons, 4 multiplications, and 4 additions).

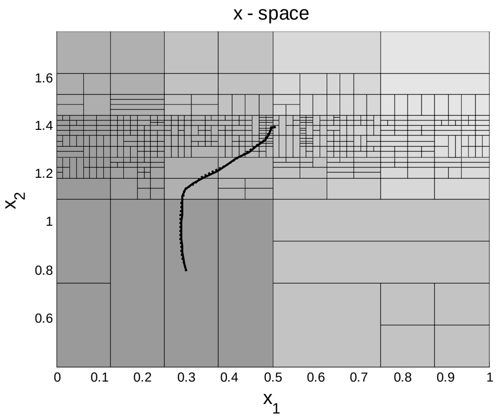

Fig. 2. State space partition of the *explicit* approximate stochastic NMPC controller and the approximate (solid curve) and the exact (dotted curve) state trajectories.

In Fig. 3, the suboptimal control functions, associated with the *explicit* approximate stochastic NMPC controller, are shown.

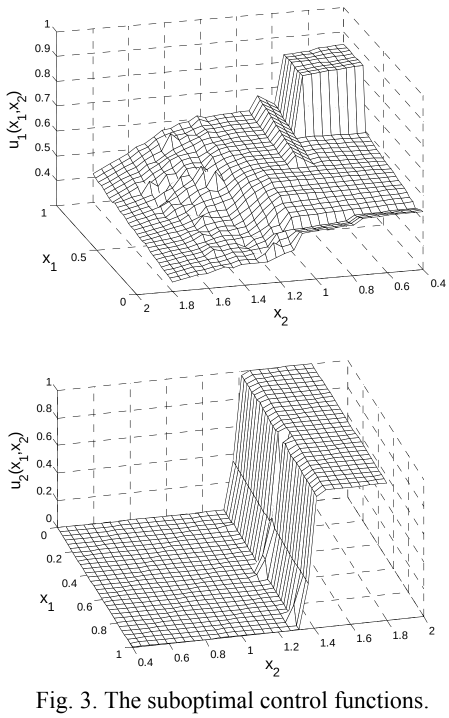

Fig. 3. The suboptimal control functions.

The performance of the closed-loop system was simulated for initial state $x(0) = [0.3\ 0.8]^T$. The response is depicted in the state space (Fig. 2), as well as trajectories in time (Fig. 4 and

Fig. 5).

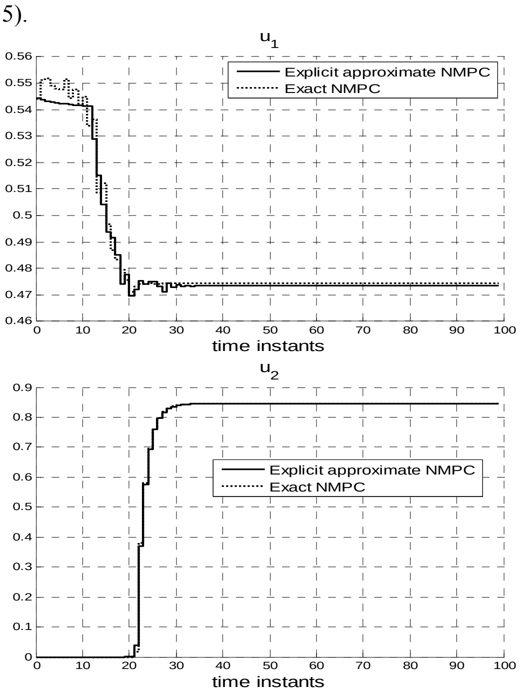

Fig. 4. The control inputs (the openness of valves V_1 and V_2).

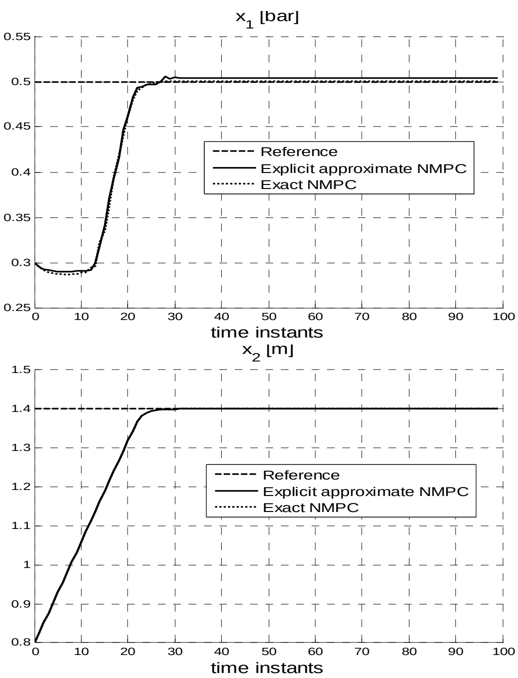

Fig. 5. The pressure x_1 and the liquid level x_2 .

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