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Vertical control of ITER plasma using explicit Model Predictive Control

Overview

- Advanced control for the vertical plasma stabilization in the ITER tokamak
- A combination of ohmic in-vessel and superconducting poloidal actuators is used
- Constrained control improves disturbance rejection subject to thermal constraints
- Constrained Linear-Quadratic optimal control
- Explicit MPC avoids on-line optimization and allows application to fast processes

Vertical stabilization of ITER plasma

A combination of actuators:

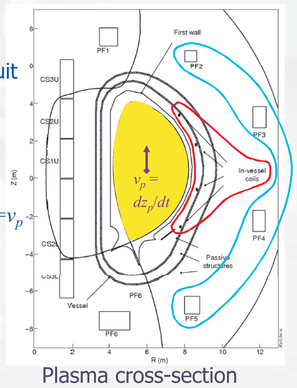
- In-vessel (IV) coils $u_1 = u_{ic}$
- Superconductive (SC) circuit VS1 (PF2-5) $u_2 = u_{VS1}$

Controlled outputs:

- IV coils current $y_1 = x_{ic}$ thermal constraint
- Plasma vertical velocity $y_2 = v_p$

Additional outputs:

- Plasma vertical position z_p
- SC circuit current i_{VS1}



Plasma cross-section

Plasma simulation model

- High-order model from first principles, CREATE-L/-NL
- Model 1 (nominal), equil. I_p=14.5 MA, β_p=0.11, l_i=0.85
- Model 2 (high gain), equil. I_p=15 MA, β_p=0.1, l_i=1.21
- Simulated VDE 10 cm: initial state of the plasma model
- ELM: simulated profiles of β_p and l_i

Static Output Feedback (SOF) control

(Ambrosino et al., 2011)

$$\mathbf{u}(t) = \mathbf{K}\mathbf{y}(t), \quad \mathbf{K} = \begin{bmatrix} 0.00285 & -2309 \\ 0.2 & 0 \end{bmatrix}$$

relatively simple, allows parameter adaptation
relatively efficient and robust performance

Constrained Linear Quadratic (CLQ) optimal control

- A systematic approach to control involving constraints
- A hybrid between LQ control and Model Predictive Control
- Increases the convergence region – larger disturbances
- Allows modification of performance by setting constraints

System: $\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k), \ \mathbf{y}(k) = \mathbf{C}\mathbf{x}(k)$

Signal constraints: $\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}, \quad \mathbf{y}_{\min} \le \mathbf{y} \le \mathbf{y}_{\max}$

Cost function: $J = \sum_{j=0}^{N-1} (\mathbf{x}_{k+j|k}^T \mathbf{Q}_x \mathbf{x}_{k+j|k} + \mathbf{u}_{k+j|k}^T \mathbf{R}_u \mathbf{u}_{k+j|k}) + \mathbf{x}_{k+N|k}^T \mathbf{P}_N \mathbf{x}_{k+N|k}$

Dual-mode ctrl.: finite horizon (trad. MPC) +terminal LQ ctrl.

Terminal cost $P_N = P_{LQ}$, assuming state feedback $\mathbf{u} = \mathbf{K}_{LQ}\mathbf{x}$

Terminal set constraint $\mathbf{x}_{k+N|k} \in X_{LQ}$
ensures optimality, stability, feasibility

Feedback parametrisation $\mathbf{u} = \mathbf{K}_{LQ}\mathbf{x} + \mathbf{f}$

Explicit (multi-parametric) MPC

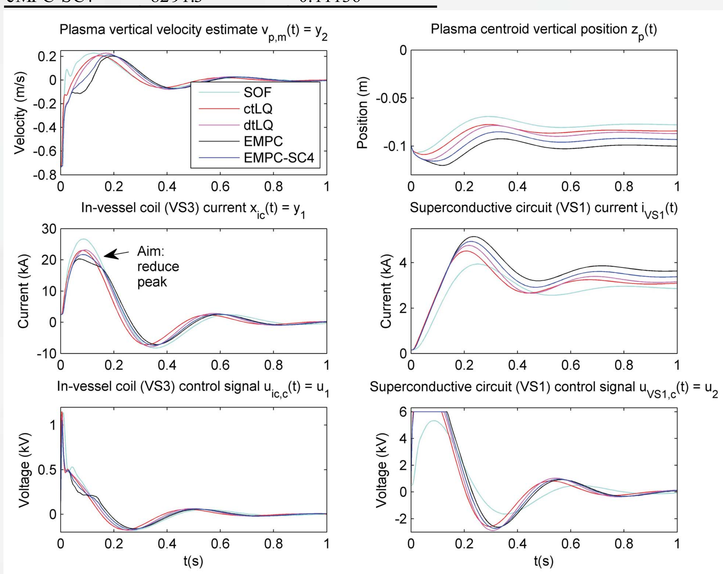
- Conventional MPC uses on-line optimization (QP) …infeasible at fast sampling rates
- Explicit MPC (S. Pistikopoulos): an equivalent parametric solution pre-computed off-line
- Moderate on-line computation: table/tree search for the active region & local control law
- Considerable off-line computation & storage requirements
- Parametric explosion: useful for small-scale problems only, not for typical industrial multivariable MPC problems
- Numerical conditioning issues
- Form of the solution:
  The control law is PieceWise-Affine function of $\mathbf{x}$
  The value function is a PieceWise-Quadratic function of $\mathbf{x}$

Performance comparison

10 cm VDE simulation, model 1

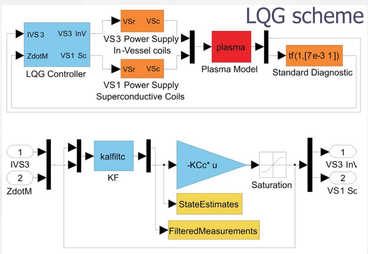
Root-Integral-Square-Error

Controller	In-vessel coil thermal load RISE($x_{ic}(t)$)	Vertical velocity regulation RISE($v_p(t)$)
SOF	9805.7	0.10786
ctLQ	8239.4	0.09625
dtLQ	8563.6	0.11658
eMPC	8061.0	0.11097
eMPC-SC4	8291.3	0.11136



Continuous-time LQ controller (ctLQ)

- Model reduction needed for eMPC and to avoid "over-fitting" (huge range, gain factor 20)
- 3rd order, Schur balanced reduction (schurmr)
- State $\mathbf{x}$ not measured, estimated using the Kalman Filter
- Saturation: avoids windup but performance not optimal



LQG controller block expanded

ctLQ tuning (nominal model + cost matrices)

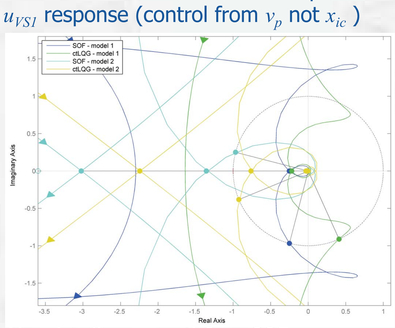
- LQG scheme may be sensitive to modelling error… Loop Transfer Recovery
- Graphical tuning environment based on linear analysis tools
- Slightly improved performance and robustness compared to SOF, due to faster u_{VS1} response (control from v_p not x_{ic})

$$\mathbf{Q}_y = \begin{bmatrix} 0.00001 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_u = \begin{bmatrix} 0.008 & 0 \\ 0 & 0.0001 \end{bmatrix}$$

$$\mathbf{Q}_{KF,y} = \mathbf{B}_r \mathbf{B}_r^T$$

$$\mathbf{R}_{KF,u} = \begin{bmatrix} 0.001 & 0 \\ 0 & 10^{-13} \end{bmatrix}$$



Nyquist diagram, ctLQ vs SOF

Discrete-time LQ controller (dtLQ)

- Model discretization using Zero-Order Hold
- Sampling time T_s = 0.005 s
  longer T_s deteriorates performance
  shorter T_s causes problems for MPC
  (prediction horizon, numerical conditioning)
- The same cost matrices as ctLQ
- A slight deterioration of performance and robustness (no longer stabilizes model 2)

CLQ controller

- Aim: reduce v_p overshoot by constraints $[-5\cdot10^4 \ \ -5]^T \le \mathbf{y} \le [2\cdot10^4 \ \ 5]^T$
- Same model, cost matrices, KF as dtLQ
- Equivalent state-cost formulation
- Terminal set constraint
- Modified Multi-Parametric Toolbox

Failure due to practical issues:

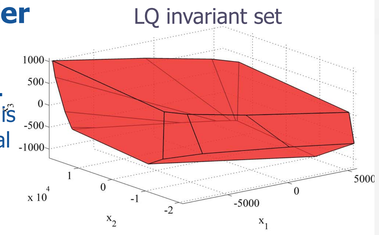
- Short achievable horizon N=2 (numerical issues)
- Too short to react to approaching $\mathbf{y}_{\max}$ constraint violation
- Initial states infeasible; LQ: u_{VS1} saturation
- Online CLQ possible with long N, but the infeasibility problem at the critical part of the partition remains
- Underestimation of v_p in model predictions

Practical eMPC controller

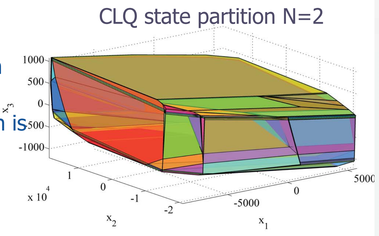
- Soft $\mathbf{y}$ constraints:
  constraint violation cost $\mathbf{S}_y = 10^{-4}\,\mathbf{I}_2$
  avoid infeasibility, more decision variables
- Control move blocking: [2 7], N=9
- Sparse placement of $\mathbf{y}$ constraints (SC):
  less decision variables,
  better numerical conditioning
- Same model, cost matrices, KF as dtLQ
  Equivalent state-cost formulation
  No terminal set constraint

CLQ controller

LQ invariant set:
LQ is feasible w.r.t. signal constraints, is constrained optimal

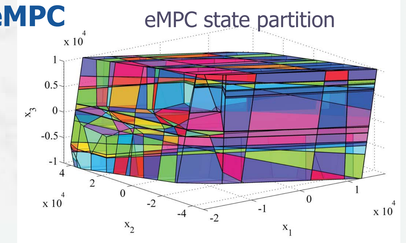


CLQ state partition N=2, 147 regions
The feasible region is increased,
but not all facets

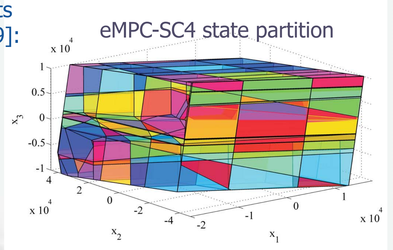


Practical eMPC

Constraints at each sample:
4826 regions



Sparse constraints at samples [1 5 9]:
531 regions



Conclusions

- Practically useful and computationally tractable eMPC controller demonstrated
- Soft constraints used to combat infeasibility
- Move blocking and sparse constraints for tractability and numerical conditioning