An algorithm to find first integrals in quasi-polynomial ODEs

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Abstract: Finding first integrals (i.e. constants of motion, hidden algebraic relationships) in ordinary differential equation systems (ODEs) is far for being a trivial problem of mathematics: for example, the differential-geometric approach needs to solve systems of partial differential equations symbolically [1]. These methods are often specialized for low dimensional ODEs, or contain heuristic steps. On the other hand, the retrieval of these hidden algebraic equations has a great – even practical – importance since the minimality of a state-space model is a necessary condition for designing state feedback type controllers for both linear and nonlinear systems [2].

An algorithm based on simple observations has been construed to find first integrals of quasi-polynomial (QP) systems. This method is capable to find any hidden algebraic relationships in QP form, which are explicit in one of their variables. This algorithm does not contain any heuristic steps, and also there is no limitation on the dimension of the ODE. It is also possible to retrieve multiple algebraic relationships if there are no cross-dependencies between them.

This algorithm has been successfully applied to a fed-batch fermentation process model and proved to be much easier to apply than solving the same problem by PDEs as in [3].

References

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