TUNING OF DECOUPLING CONTROLLER BY USING MOMI METHOD

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Abstract: A large majority of existing tuning methods for multivariable PI controllers require good process model in order to achieve satisfactory tuning. Modelling and identification of an accurate process model is time consuming and requires extensive experimentation. In this paper, a new parameterisation of the process model based on multiple integration of the process step-response, is proposed. It is shown that the proposed parameterisation can be successfully used for tuning parameters of decoupling controllers. Experiments taken on two process models showed that the proposed method performs better than certain other tuning methods requiring more demanding modelling and identification.

Keywords: multivariable systems, PID, tuning, decoupling

1. INTRODUCTION

Many systems in chemical and process industry are multivariable (MV). In most cases cross-coupling between inputs and outputs is low. Therefore conventional univariable (SISO) controllers can be successfully applied. However, if multivariable systems exhibit stronger cross-coupling between process inputs and outputs, multivariable controllers should be applied in order to achieve satisfactory performance.

The most common types of controllers are multivariable controllers (Fig. 1) and decoupling controllers (Fig. 2). They usually consists of PI or PID controllers since their structure is relatively simple and offer a good trade-off between performance and robustness.

Some of the most recent approaches tend to make tuning procedure simple and user-friendly. Wang et al. [10] proposed auto-tuning method for multivariable PID controllers based on relay excitation. The process model is derived from the frequency response at two points by using biased relay feedback. The multivariable PID controller parameters are then calculated according to the selected gain and phase margins. Wang et al. [11] have presented an approach for tuning two-inputs-two-outputs (TITO) systems by using decoupling controllers. Controller and decoupler tuning was based on the 1st order and the 2nd order plus dead-time models which were derived from the process step-response. The gain and phase margins and additional tuning factor α need to be specified.

Recently, a new method for tuning multivariable processes, which is based on multiple integration of process responses, has been developed [7]. The main idea was to simplify the procedure for calculating the parameters of a multivariable PI controller, originally proposed by Lieslehto [3]. Namely, the explicit process modelling and identification phase was replaced by performing multiple integration of the process steady-state change. Thus, the multivariable process is parameterised using these multiple integrals instead of the parameters of the process transfer functions. Such parameterisation is simpler since it can use any kind of open-loop change of the process steady-state.

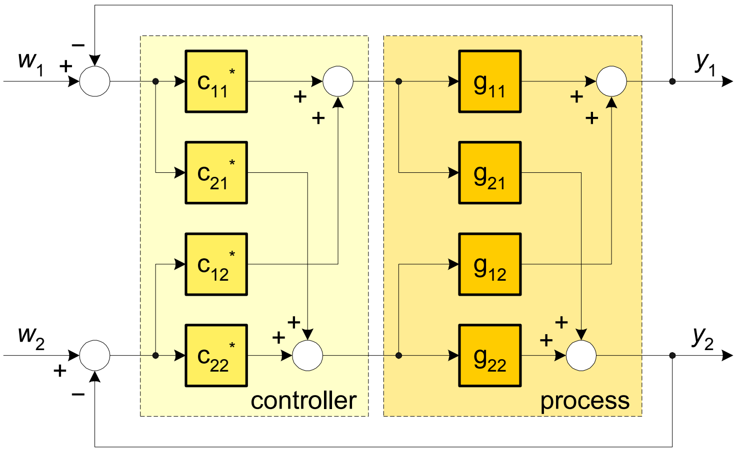

Fig. 1. Multivariable (TITO) process with multivariable controller.

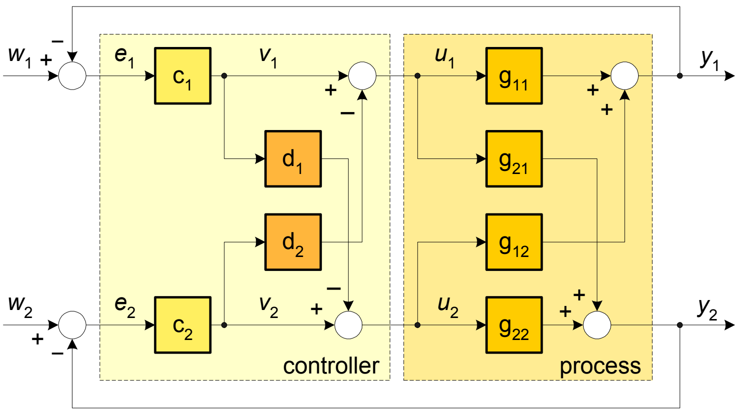

Fig. 2. Multivariable (TITO) process with decoupling controller (controllers c_1 and c_2 and decouplers d_1 and d_2).

In this paper, the method is extended to the decoupling controllers. The advantage of using a decoupling controller over a multivariable PI controller is that decoupling and tuning are separate tasks. Therefore, the calculation of decoupling controller parameters becomes more straightforward and in some cases decoupling becomes more efficient.

2. DECOUPLING CONTROLLER

Fig. 2 depicts a simple two-inputs-two-outputs (TITO) multivariable system. A decoupling controller consists of two SISO controllers (c_1 and c_2) and two decouplers (d_1 and d_2).

The first control goal is to decouple the multivariable system by means of decouplers d_1 and d_2. This can be done by choosing the following transfer functions for decouplers:

$$d_1(s) = \frac{g_{21}(s)}{g_{22}(s)},$$
$$d_2(s) = \frac{g_{12}(s)}{g_{11}(s)} \quad (1)$$

If expression (1) holds, cross-interactions between inputs v_1 and v_2 an outputs y_1 and y_2 do not exist. In this case the process outputs are:

$$y_1(s) = c_1(s) g_1(s) e_1(s)$$
$$y_2(s) = c_2(s) g_2(s) e_2(s). \quad (2)$$

where

$$g_1(s) = g_{11}(s) - d_1(s) g_{12}(s)$$
$$g_2(s) = g_{22}(s) - d_2(s) g_{21}(s) \quad (3)$$

The second control goal is to tune controllers c_1 and c_2 for the processes g_1 and g_2, respectively.

Unfortunately, explicit transfer functions of the sub-processes are rarely known in practice. In most cases, only the simplest measurements like process step responses are available. In such cases the tuning of decoupling controller should be suitably modified. This can be done by using the multiple integration method [5,6] for the calculation of decoupler model parameters (d_1 and d_2) and the Magnitude-Optimum-Multiple-Integration (MOMI) tuning method [6,7,9] for tuning controllers c_1 and c_2.

3. TUNING OF DECOUPLING CONTROLLER PARAMETERS

The following areas (A_0..A_k) can be expressed by integrating the process open-loop step response ($y(t)$), after applying the step-change ΔU at the process input at t=0 [5,8]:

$$A_0 = y_0(\infty)$$
$$\vdots$$
$$A_k = y_k(\infty) \quad (4)$$

where

$$y_0(t) = \frac{y(t) - y(0)}{\Delta U}$$
$$\vdots$$
$$y_k(t) = \int_0^t \left[A_{k-1} - y_{k-1}(\tau)\right] d\tau \quad (5)$$

Note that A_0 represents the steady-state gain of the process.

If the process is known and given by the following transfer function:

$$G_P(s) = K_{PR} \frac{1 + b_1 s + b_2 s^2 + \cdots + b_m s^m}{1 + a_1 s + a_2 s^2 + \cdots + a_n s^n} e^{-sT_{del}}, \quad (6)$$

then, it can be parametrised by the following characteristic areas [5,6]:

$$
\begin{aligned}
A_0 &= K_{PR} \\
A_1 &= K_{PR}\left(a_1 - b_1 + T_{del}\right) \\
A_2 &= K_{PR}\left[b_2 - a_2 - T_{del} b_1 + \frac{T_{del}^{\;2}}{2!}\right] + A_1 a_1 \\
&\;\vdots \\
A_k &= K_{PR}\left(\left(-1\right)^{k+1}\left(a_k - b_k\right) + \sum_{i=1}^{k}\left(-1\right)^{k+i}\frac{T_{del}^{\;i} b_{k-i}}{i!}\right) + \\
&+ \sum_{i=1}^{k-1}\left(-1\right)^{k+i-1} A_i a_{k-i}
\end{aligned}
\qquad (7)
$$

Areas (7) can also be measured in time-domain from the process steady-state change [8].

Based on measured areas, the process transfer function parameters can be estimated from (7). The number of estimated process transfer function parameters equals the number of measured areas. The second-order process model with two poles and one zero can be calculated from the measured areas as follows:

$$
\begin{aligned}
K_{PR} &= A_0 \\
a_1 &= \frac{A_0 A_3 - A_1 A_2}{A_0 A_2 - A_1^{\;2}} \\
a_2 &= \frac{A_1 a_1 - A_2}{A_0} \\
b_1 &= a_1 - \frac{A_1}{A_0}
\end{aligned}
\qquad (8)
$$

Decouplers d_1 and d_2 (1) can also be calculated from the measured areas of the sub-processes g_{11} to g_{22}. The areas of decoupler d_1 can be calculated by using the following expression:

$$
A_{n_{d1}} = \sum_{j=0}^{n}\left(A_{j_{21}} A_{(n-j)_m}\right), \qquad (9)
$$

where

$$
\begin{aligned}
A_{0_m} &= \frac{1}{A_{0_{22}}} \\
A_{1_m} &= -\frac{A_{1_{22}}}{A_{0_{22}}^{\;2}} \\
A_{2_m} &= \frac{A_{1_{22}}^{\;2}}{A_{0_{22}}^{\;3}} - \frac{A_{2_{22}}}{A_{0_{22}}^{\;2}} \\
A_{3_m} &= -\frac{A_{1_{22}}^{\;3}}{A_{0_{22}}^{\;4}} + \frac{2A_{1_{22}} A_{2_{22}}}{A_{0_{22}}^{\;3}} - \frac{A_{3_{22}}}{A_{0_{22}}^{\;2}}
\end{aligned}
\qquad (10)
$$

Indexes 21 and 22 represent the areas of the sub-processes g_{21} and g_{22}, respectively. For decoupler d_2, index d_1 in expression (9) should be replaced by d_2 while indexes 21 and 22 in expression (10) by 12 and 11, respectively. From obtained areas (9), the second-order decouplers (d_1 and d_2) can be calculated from (8).

In some cases, the obtained decoupler is not physically realisable (the high-frequency gain is infinite or too high). In this case, the first-order filter should be added to decoupler as shown in Fig. 3. As a rule of thumb, the following filter time constant is chosen:

$$\begin{aligned} T_f &= 0; \ \text{if } |b_1| \le 4|a_1| \\ T_f &= 0.25|b_1| - |a_1|; \ \text{if } |b_1| > 4|a_1| \end{aligned} . \tag{11}$$

If damping of decoupler poles is too low, the parameter a_2 should be limited. As a rule of thumb, the following limitation has been chosen:

$$a_2 \le 0.5 a_1^2 . \tag{12}$$

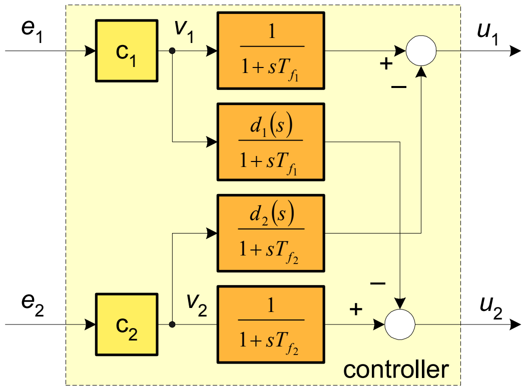

Fig. 3. Filtering decouplers d_1 and d_2 by the first-order filters.

If a_2 has been modified according to expression (12), The parameter a_1 should be re-calculated from (7):

$$a_1 = \frac{A_1 + \sqrt{A_1^2 - 2A_0A_2}}{2A_0} . \tag{13}$$

After obtaining decouplers, the controllers c_1 and c_2 should be tuned. The PI controller structure has been chosen:

$$v(s) = \left(K + \frac{K_i}{s}\right)e(s), \tag{14}$$

where e(s) is the control error and v(s) is controller output (see Fig. 2). The calculation of the controllers parameters can be performed by using the MOMI tuning method [6-9]. The method requires only the measured areas of the process g_1 or g_2 (3) to calculate controller parameters:

$$\begin{aligned} K &= \frac{A_3}{2(A_1A_2 - A_0A_3)} \\ K_i &= \frac{KA_0 + 0.5}{A_1} \end{aligned} . \tag{15}$$

The areas of the processes g_1 and g_2 (3) can be expressed by areas of sub-processes g_{11} to g_{22} and areas of decouplers d_1 and d_2 (9) in the following way:

$$A_{n_{g1}} = A_{n_{11}} - \sum_{j=0}^{n} \left(A_{j_{12}} A_{(n-j)_{d1}} \right)$$
$$A_{n_{g2}} = A_{n_{22}} - \sum_{j=0}^{n} \left(A_{j_{21}} A_{(n-j)_{d2}} \right) \qquad (16)$$

The parameters of controllers c_1 and c_2 are then calculated by substituting areas A_n in (15) with A_{ng1} and A_{ng2} (16).

If the filter (11) has been added to decouplers, the areas A_{ng1} and A_{ng2} should be additionally modified as follows:

$$A'_{n_{g1}} = \sum_{j=0}^{n} \left(A_{j_{g1}} T_{f_1}^{\ n-j} \right)$$
$$A'_{n_{g2}} = \sum_{j=0}^{n} \left(A_{j_{g2}} T_{f_2}^{\ n-j} \right) \qquad (17)$$

The complete decoupling controller tuning procedure therefore proceeds as follows:

1. Apply the step change ΔU at the first process input and measure all process outputs. The responses of sub-processes g_{11} and g_{21} are thus obtained. Repeat the procedure for the second input and obtain the responses of sub-processes g_{12} and g_{22}.
2. Obtain the process gain $K_{PR_{ij}}$ ($A_{0_{ij}}$) and three areas ($A_{1_{ij}}$, $A_{2_{ij}}$, and $A_{3_{ij}}$) by using numerical integration, according to expression (4).
3. Calculate decouplers' areas from (9).
4. Calculate decouplers' transfer functions (d_1 and d_2 in Fig. 2) from obtained areas in step 3 by using expression (8).
5. If required, add filters and/or modify decouplers transfer functions according to expressions (11) to (13).
6. Obtain characteristic areas of the processes g_1 and g_2 from (16) and (17).
7. Calculate the PI controller parameters (c_1 and c_2 in Fig. 2) from obtained areas in the previous step by using expression (15).

The matlab toolset which performs the calculation of decoupling controller parameters can be downloaded from [9].

Note: If the calculated gain K in (15) is too high (especially for the lower-order processes), it can be limited to some arbitrary value. The integral gain should be then re-calculated from (15) by using the new value of the controller gain.

4. EXAMPLES

Two examples are performed to illustrate the proposed design of decoupling controller.

Case 1

The first example is performed on the multivariable model given by Menani and Koivo [4]:

$$\mathbf{G}(s) = \frac{1}{(1+0.1s)(1+0.2s)^2} \begin{bmatrix} \frac{0.5}{1+0.1s} & -1 \\ 1 & \frac{2.4}{1+0.5s} \end{bmatrix}. \tag{18}$$

According to the proposed tuning procedure, two step-changes are performed at two process inputs.

At first, the process steady-state gains are measured as: $K_{PR_{11}}$=0.5, $K_{PR_{12}}$=-1, $K_{PR_{21}}$=1, $K_{PR_{22}}$=2.4. The areas A_1 to A_3 are then calculated from the step-responses obtained for all sub-processes by using numerical integration, as explained in section 3. The calculated values are for g_{11}: A_1=0.3, A_2=0.115, A_3=0.036, for g_{12}: A_1=-0.5, A_2=-0.17, A_3=-0.049, for g_{21}: A_1=0.5, A_2=0.17, A_3=0.049, and for g_{22}: A_1=2.4, A_2=1.608, A_3=0.9216.

The following decouplers were calculated from (8):

$$\begin{aligned} d_1(s) &= 0.417(1+0.5s) \\ d_2(s) &= -2(1+0.1s) \end{aligned}. \tag{19}$$

Since the calculated decouplers cannot be realised in practice (due to pure derivative terms), the following filters are added, according to expression (11):

$$T_{f_1} = 0.125;\ T_{f_2} = 0.025\ . \tag{20}$$

The PI controller parameters are then calculated from (15) to (17):

$$\begin{aligned} K_1 &= 0.906;\ K_{i_1} = 3.209 \\ K_2 &= 0.281;\ K_{i_2} = 0.524 \end{aligned}. \tag{21}$$

The resulting closed-loop step-responses on the reference change at both controller inputs are given in Fig. 4. The response is compared to that obtained by Menani and Koivo [4]. Their multivariable PI controller was derived by use of the Maciejowski MIMO controller design technique, on the process model obtained by using the relay excitation method.

The resulting closed-loop performance of the proposed method is very good, since it is relatively fast and ideally decoupled.

Case 2

The second example is performed on the process model of a methanol-water distillation column given by Ho *et al*. [2] and Menani and Koivo [4]:

$$\mathbf{G}(s) = \begin{bmatrix} \frac{12.8e^{-s}}{1+16.7s} & \frac{-18.9e^{-3s}}{1+21s} \\ \frac{6.6e^{-7s}}{1+10.9s} & \frac{-19.4e^{-3s}}{1+14.4s} \end{bmatrix}. \tag{22}$$

The same tuning procedure is performed as in the previous case.

The process steady-state gains are measured as: $K_{PR_{11}}$=12.8, $K_{PR_{12}}$=-18.9, $K_{PR_{21}}$=6.6, $K_{PR_{22}}$=-19.4. The calculated values of areas are for g_{11}: A_1=226.56, A_2=3.79e3, A_3=6.329e4, for g_{12}: A_1=-453.6, A_2=-9.6106e3, A_3=-2.0191e5, for g_{21}: A_1=118.14, A_2=1.4494e3, A_3=1.618e4, and for g_{22}: A_1=-337.56, A_2=-4.9482e3, A_3=-7.134e4.

The following decouplers were calculated from (8):

$$d_1(s) = \frac{-0.34(1+12.2s)}{(1+12.7s+50.5s^2)}$$
$$d_2(s) = \frac{-1.48(1+17.4s)}{(1+23.7s+48.5s^2)} . \quad (23)$$

The PI controller parameters are calculated from (15) and (16):

$$K_1 = 0.064;\ K_{i_1} = 0.013$$
$$K_2 = -0.052;\ K_{i_2} = -0.0098 . \quad (24)$$

The proposed decoupling PI controller is compared with two diagonal PI controllers obtained by Ho *et al*. [2] The controllers were calculated according to the given phase and gain margin specifications. The process model was obtained using a least-squares process identification.

The closed-loop time responses for both cases are given in Fig. 5. It is shown that the proposed method again results in a relatively good closed-loop performance. On the other hand, the process responses obtained by Ho *et al*. [2] are faster, but not so very well decoupled.

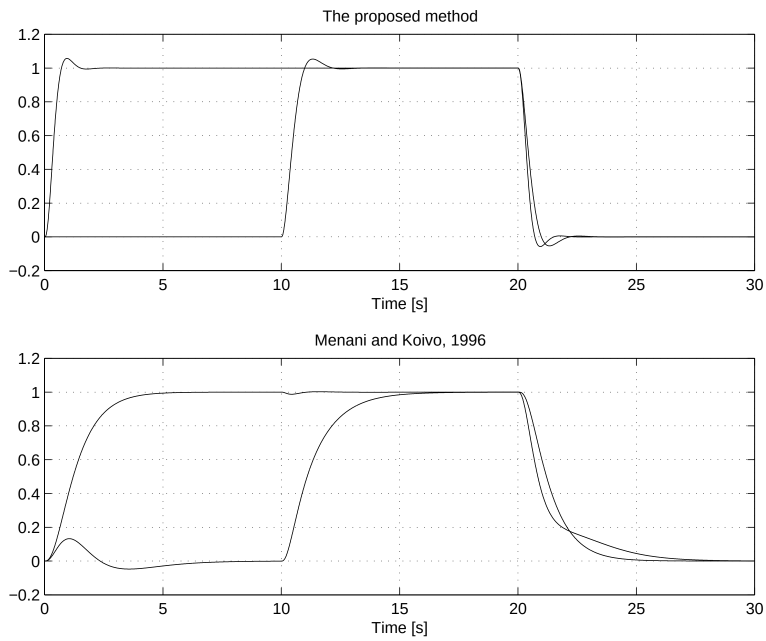

Fig. 4. Process closed-loop time responses when using the proposed decoupling controller (upper figure), and when using the setting proposed by Menani and Koivo (lower figure).

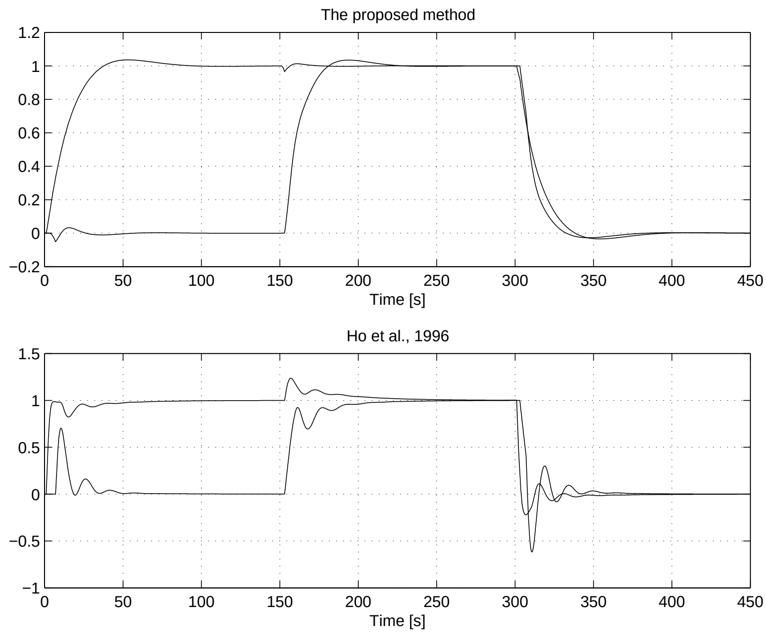

Fig. 5. Process closed-loop time responses when using the proposed decoupling controller (upper figure), and when using the setting proposed by Ho et al. (lower figure).

5. CONCLUSIONS

In this paper, a new simple tuning method is proposed. The novelty of the proposed approach is to use alternative parameterisation of the process (calculation of areas), which does not require explicit process modelling and substantially simplifies the identification stage (the process open-loop step response suffice). Calculation of areas and the remaining tuning procedure can also be performed automatically, since all the required calculations are not numerically demanding.

Simulation results showed that the performance of the proposed method is quite good when compared with some other existing tuning methods. Some of the existing methods require a more demanding modelling and identification stage. However, low-frequency noise and process non-linearities could affect the accuracy of calculated areas, the multivariable process must consist of stable sub-processes (all poles on the left half-plane), and degradation of the closed-loop performance may be expected if the sub-process transfer functions are of different orders of magnitude.

The main emphasis of this paper was on simple tuning procedure for decoupling controller design. Therefore, decoupling controller structure was chosen as simple as possible. However, it should be stressed that the closed-loop performance could be additionally improved by using PID instead of PI controllers [10,11] or by using different transfer function for decouplers d_1 and d_2 (e.g. first-order transfer function with time delay in Case 2).

6. ACKNOWLEDGEMENTS

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